Information Economics
Suggested Solution for Problem Set 3

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1. (a) Let the decision variables be

$$x_i = \text{price of product } i,\ i = A, B.$$

The problem can then be formulated as

$$\begin{aligned}
\max \quad & 1000x_A + 1500x_B \\
\text{s.t.} \quad & 10 - x_A \geq 0 && \text{(IR-1)} \\
& 15 - x_B \geq 0 && \text{(IR-2)} \\
& 10 - x_A \geq 8 - x_B && \text{(IC-1)} \\
& 15 - x_B \geq 12 - x_A. && \text{(IC-2)}
\end{aligned}$$

The objective function maximizes the total sales revenue because group 1 members purchase A and group 2 members purchase B. Constraint (IR-1) ensures that a group 1 member is willing to buy product A. Constraint (IR-2) ensures that a group 2 member is willing to buy product B. Constraint (IC-1) ensures that a group 1 member prefers product A. Constraint (IC-2) ensures that a group 2 member prefers product B.

(b) First, note that (IR-2) is redundant because

$$15 - x_B \geq 12 - x_A \geq 10 - x_A \geq 0,$$

where the first inequality is (IC-2) and the last inequality is (IR-1). Once we remove (IR-2), we can show that (IC-2) is binding at any optimal solution. Suppose this is not the case, we will increase x_B for a sufficiently small amount without violating any constraint. We then have $x_B = x_A + 3$, which implies that (IC-1) is satisfied by any optimal solution because

$$8 - x_B = 5 - x_A \leq 10 - x_A.$$

Once we remove (IC-1), it is clear that (IR-1) must be binding at any optimal solution, so $x_A = 10$ and $x_B = 13$. These are the optimal prices.

2. (a) $\theta \in \{r_L, r_H\}$ is the retailer's type. $v(q)$ is the expected sales volume given the inventory level q, which is

$$\int_0^q x f(x) dx + \int_q^1 q f(x) dx = q - \frac{1}{2}q^2.$$

Within $[0, 1]$, it is clear that $v'(q) > 0$ and $v''(q) < 0$, and thus $v(q)$ is strictly increasing and strictly concave in the domain of interest.

(b) Our formula $\theta_i v'(q_i^{FB}) = c$ translates to $r_i(1 - q_i^{FB}) = c$, i.e., $q_i^{FB} = 1 - \frac{c}{r_i}$. The associated transfer is $t_i^{FB} = r_i v(q_i^{FB}) = \frac{1}{2r_i}(r_i^2 - c^2)$.

(c) The problem can be formulated as

$$\begin{aligned}
\max \quad & \beta(t_L - cq_L) + (1 - \beta)(t_H - cq_H) \\
\text{s.t.} \quad & r_L\left(q_L - \frac{1}{2}q_L^2\right) - t_L \geq r_L\left(q_H - \frac{1}{2}q_H^2\right) - t_H && \text{(IC-L)} \\
& r_H\left(q_H - \frac{1}{2}q_H^2\right) - t_H \geq r_H\left(q_L - \frac{1}{2}q_L^2\right) - t_L && \text{(IC-H)} \\
& r_L\left(q_L - \frac{1}{2}q_L^2\right) - t_L \geq 0 && \text{(IR-L)} \\
& r_H\left(q_H - \frac{1}{2}q_H^2\right) - t_H \geq 0 && \text{(IR-H)}
\end{aligned}$$

(d) Because $\frac{r_H - r_L}{r_H} = \frac{1}{5} < \frac{1}{2}$, we have

$$r_H v'(q_H^*) = r_H(1 - q_H^*) = c \qquad \Longleftrightarrow \qquad q_H^* = 1 - \frac{2}{10} = \frac{4}{5}$$

and

$$r_L v'(q_L^*) = r_L(1 - q_L^*) = c\left(\frac{1}{1 - \frac{1-\beta}{\beta}\frac{r_H - r_L}{r_L}}\right) \qquad \Longleftrightarrow \qquad q_L^* = 1 - \frac{8/3}{8} = \frac{2}{3}.$$

The associated transfers are $t_L^* = 8(\frac{2}{3} - \frac{2}{9}) = \frac{32}{9}$ and $t_H^* = 10(\frac{12}{25} - \frac{4}{9}) + \frac{32}{9} = \frac{176}{45}$.

(e) We have $\frac{2}{3} < \frac{4}{5}$, which means $q_L^* < q_H^*$. Moreover, we have $\frac{2}{3} < 1 - \frac{2}{8} = \frac{3}{4}$, which mean $q_L^* < q_L^{FB}$.