1. (The Leibniz integral rule) Applying the Leibniz integral rule to the following calculations:

   (a) $\frac{d}{dy}\int_0^{20} xydx.$

   (b) $\frac{d}{dy}\int_0^{y} x^2dx.$

   (c) $\frac{d}{dy}\int_0^{y} xydx.$

   (d) $\frac{d}{dy}\int_0^{\sin y} xydx.$

   (e) $\frac{d}{dy}\int_{\ln y}^{\sin y} xydx.$

2. (Newsvendor) Find the optimal order quantity q^* when the random demand follows a uniform distribution between 0 and b, the unit production cost is c, and the unit retail price is p. How do b, c, and p affects q^*? Intuitively explain why.

3. (Returns) Consider a retailer under a partial-return contract with a return credit $0 < r \leq w$, where w is the wholesale price, and the allowable return rate $R = \frac{1}{2}$.

   (a) Formulate the retailer's optimization problem.

   (b) Find the optimal order quantity q^* when the random demand follows a uniform distribution between 0 and b, unit production cost is c, and the unit retail price is p.

   (c) Find the coordinating return credit r^*.

   (d) How do b, c, p, and w affects r^*? Intuitively explain why.

4. (Revenue sharing) In a revenue sharing contract (w, ϕ) in a manufacturer-retailer relationship, the manufacturer charges the retailer $w \geq 0$ per unit of product and the retailer gives ϕs to the manufacturer if the sales revenue is s for some $\phi \in [0, 1]$. Suppose the retailer is a newsvendor facing a random demand $D \sim F, f$, the unit production cost is c, and the unit retail price is p.

   (a) How is the wholesale contract a special case of this?

   (b) Formulate the retailer's ordering problem given w and ϕ. Then find the retailer's optimal order quantity $q^*(w, \phi)$ as a function of w and ϕ.

   (c) According to Part (b), what kind of revenue sharing contracts coordinate the channel?

   (d) Can revenue sharing contracts achieve arbitrary profit splitting?

5. (Channel rebates) In a channel rebate contract (w, T, s) between a manufacturer and a retailer, the manufacturer charges the retailer $w \geq 0$ per unit of product and gives the retailer a sales bonus $s \geq 0$ for each unit of sales above $T \geq 0$. A channel rebate contract is called a linear rebate contract if $T = 0$ or a target rebate contract if $T > 0$. Suppose the retailer is a newsvendor facing a random demand $D \sim F, f$, the unit production cost is c, and the unit retail price is p.

   (a) How is the wholesale contract a special case of this?

   (b) Find the retailer's expected profit as a function of q, the order quantity, given w, T, and s. Note that for $q < T$ and $q \geq T$, the expected profit functions are different.

(c) Suppose that $T = 0$ and the manufacturer is restricted in using linear rebates. What is the retailer's optimal order quantity $q^*(w, s)$ as a function of w and s?

(d) According to Part (c), what kind of channel rebate contracts coordinate the channel?

(e) Suppose that s is bounded above by p. Can channel rebate contracts achieve arbitrary profit splitting?

6. (Comparisons of returns, revenue sharing, and channel rebates) While the return contracts, revenue sharing contracts, and channel rebate contracts are all coordinating, is there any reason to prefer one to the other ones?