Information Economics

Fall 2013

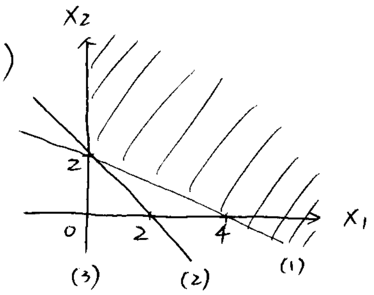
Suggested Solution to Homework 1

1. (a) 22 (b) $\nabla f(x) = \begin{bmatrix} 12x_1^3 + 4x_2^2 \\ 8x_1x_2 - 2x_2 \end{bmatrix}$, $\nabla^2 f(x) = \begin{bmatrix} 36x_1^2 & 8x_2 \\ 8x_2 & 8x_1 - 2 \end{bmatrix}$

(c) $2e^{2x}\left[\frac{x}{x^2+2} + \ln(x^2+2)\right]$ (d) $8x - 12$.

2. (a) $\frac{64}{3}$ (b) $\frac{1}{2}e^{2x}$ (c) $\frac{1}{3}x_1x_2^3 + x_2 \sin x_1$ (d) $x^2 + 2x - 3$.

3. (a)



(b) $(0, 2)$

(c) Constraint 2 : $x_1 + x_2 \geq 2$.

4. (a) $\Pr(X > \frac{1}{4}) = \int_{\frac{1}{4}}^{\infty} 5e^{-5x}\,dx = e^{-\frac{5}{4}} \approx 0.2865$. (b) 10.

5. Let $x_1, x_2 \in X \cap Y$ and $\lambda \in [0, 1]$, then we have $\lambda x_1 + (1-\lambda)x_2 \in X$ because X is convex and $\lambda x_1 + (1-\lambda)x_2 \in Y$ because Y is convex. It then follows that $\lambda x_1 + (1-\lambda)x_2 \in X \cap Y$ and thus $X \cap Y$ is convex.

6. Let $x_1, x_2 \in F$ and $\lambda \in [0, 1]$, then we have

$$\begin{aligned} h(\lambda x_1 + (1-\lambda)x_2) &= f(\lambda x_1 + (1-\lambda)x_2) + g(\lambda x_1 + (1-\lambda)x_2) \\ &\leq \lambda f(x_1) + (1-\lambda) f(x_2) + \lambda g(x_1) + (1-\lambda) g(x_2) \\ &= \lambda h(x_1) + (1-\lambda) h(x_2), \end{aligned}$$

so h is convex.

7. (a) $\{1\}$ (b) $\{-2, 0\}$ (c) 3 (d) $\{-1\}$ (e) $\{-1\}$

8 (a) $\because f(x) \geq 0 \ \forall x$ and $\int_0^1 f(x)\,dx = x^2\Big|_0^1 = 1$, f is a pdf.

(b) The probability is $\int_p^1 2x\,dx = x^2\Big|_p^1 = 1 - p^2$. The expectation is $a(1-p^2)$.

(c) $\pi'(p) = D(p) + D'(p)(p-c)$. $\pi''(p) = 2D'(p) + D''(p)(p-c)$.

(d) We know $D'(p) \leq 0$ and $D''(p) \leq 0$, which imply $\pi'(p) \leq 0$ as $p \geq c$.

Therefore, $\pi(p)$ is concave.

(e) If $g(\cdot)$ is nondecreasing, $g(p_1) = c_1$, and $g(p_2) = c_2$, then $c_2 > c_1$

$\Rightarrow p_2 \geq p_1$, which means increasing c weakly increases the p s.t. $g(p) = c$.

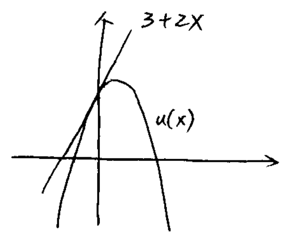
(In fact, you should show that $g(p) = c$ has a unique solution. Try it!)

To see that $g(\cdot)$ is nondecreasing, note that $g'(p) = 2 - \dfrac{D''(p)\,D(p)}{[D'(p)]^2}$.

Because $D''(p) \leq 0$ and $D(p) \geq 0$, $g'(p) \geq 0$, so $g(\cdot)$ is nondecreasing.

9 (a) Applying the RHS of the lemma to $u(x) = -x^2 + 2x + 3$

and $x_0 = 0$, we get $3 + (-2x+2)\big|_{x=0}(x) = 3 + 2x$.



Because $-x^2 \leq 0$, $u(x) = -x^2 + 2x + 3 \leq 2x + 3 \ \forall x \in \mathbb{R}$.

(b) Because $u(\cdot)$ is concave, we have for any $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$,

$u(\lambda x_1 + (1-\lambda)x_2) \geq \lambda u(x_1) + (1-\lambda) u(x_2)$

$\Leftrightarrow u(x_2 + \lambda(x_1 - x_2)) \geq u(x_2) + \lambda(u(x_1) - u(x_2))$

$\Leftrightarrow \dfrac{u(x_2 + \lambda(x_1 - x_2)) - u(x_2)}{\lambda} \geq u(x_1) - u(x_2)$ (now take $\lim\limits_{\lambda \to 0}$ at both sides)

$\Leftrightarrow u'(x_2)(x_1 - x_2) \geq u(x_1) - u(x_2) \Leftrightarrow u(x_1) \leq u(x_2) + u'(x_2)(x_1 - x_2)$.

Now label x_1 as x and x_2 as x_0 and we are done.

(c) $\int_{-\infty}^{\infty} f(x)\,u(x)\,dx \leq \int_{-\infty}^{\infty} f(x)\,u(\mu)\,dx + \int_{-\infty}^{\infty} f(x)\,u'(\mu)(x-\mu)\,dx$

$\Rightarrow \mathbb{E}[u(X)] \leq u(\mu) + u'(\mu)\left\{\mathbb{E}[X] - \mathbb{E}[X]\right\}$

$\Rightarrow \mathbb{E}[u(X)] \leq u(\mathbb{E}[X])$.