# Information Economics, Spring 2013
# Homework 4a

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**Note 1.** This homework is due ***8:30 am, October 28, 2013***. Please submit a hard copy into the instructor's mail box. As each team only needs to submit one copy, please indicate the names and student IDs of all team members on the first page.

**Note 2.** Each team must have ***exactly three*** students unless a special approval is obtained.

1. (20 points; 4 points each) Consider a manufacturer-retailer relationship in a supply chain. The manufacturer produces a product and sell it to the retailer. The retailer is a newsvendor facing random market demand $D \sim \text{Uni}(0,1)$. The unit retail price is fixed to $r$, which is public to everyone. However, the retailer may be either efficient or inefficient: An efficient retailer sells a product with a unit retail cost $d_1$ while an inefficient one does so with a unit retail cost $d_2$, where $d_1 < d_2$. Therefore, the unit "net sales revenues" of an efficient retailer and an inefficient retailer are defined to be $r_H = r - d_1$ and $r_L = r - d_2$. Naturally, $r_L < r_H$. The retailer's retail cost is his private information. The manufacturer believes that the retailer is inefficient with probability $\beta$ or efficient with probability $1 - \beta$. Each player acts to maximize her/his expected profit. Before the selling season starts, the manufacturer offers the retailer a menu of two contracts, $\{(q_L, t_L), (q_H, t_H)\}$, where $(q_i, t_i)$ is intended for the type-$i$ retailer, $i \in \{L, H\}$. If the retailer selects $(q_i, t_i)$, he obtains $q_i$ units from the manufacturer by paying $t_i$. He then faces a typical newsvendor situation with $q_i$ units of inventory at the beginning of the selling season.

   (a) In our two-type monopoly pricing model, we assumed that the agent's utility function is $v(q, t, \theta) = \theta v(q) - t$, where $\theta$ is the agent's type and $v(\cdot)$ is strictly increasing and strictly concave. For the retailer in this problem, what is $\theta$? What is $v(q)$? Write down the closed-form expression of $v(q)$ as a function of $q$. Show that it is strictly increasing and strictly concave in the domain of interest.

   (b) By assuming complete information, find the first-best menu for the two types of retailers.

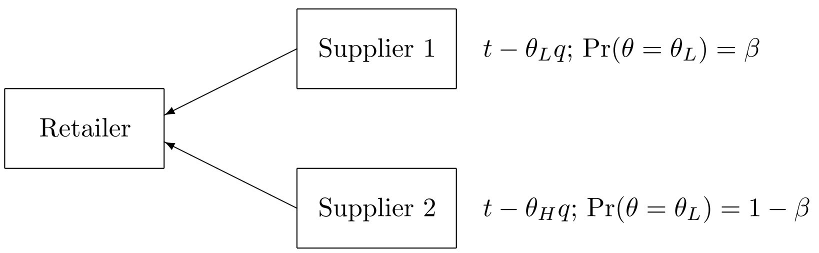
   (c) Suppose there is information asymmetry, formulate the manufacturer's contract design problem whose solution is the second-best menu.

   (d) Continue from Part (c). Suppose $c = 2$, $r_L = 8$, $r_H = 10$, and $\beta = \frac{1}{2}$. Find the manufacturer's second-best menu.

   **Note.** Plugging in numbers into the formula we derived on slides is fine.

   (e) Continue from Part (d). Show that the low-type order quantity is lower than the high-type order quantity. Then show that the low-type order quantity is lower than the efficient level.

2. (20 points) A retailer is buying a product from a supplier, which may produce the product at a unit cost $\theta_L$ with probability $\beta$ or $\theta_H$ with probability $1 - \beta$. Assume $\theta_L < \theta_H$ and such a cost is the supplier's private information. See the figure below for an illustration.




We refer to a pair of transfer and quantity $(t, q)$ as a contract. For example, if a supplier chooses $(t, q) = (500, 5)$, the retailer will buy 5 units from the supplier and pays \$500 to the supplier.

Therefore, if a type-$i$ supplier chooses a contract $(t, q)$, his profit is $t - \theta_i q$. The retailer generates sales revenues by selling those products she obtains. Assume that the sales revenue is a function of the number of products she has and is denoted as $v(q)$, which is strictly increasing and strictly concave. Therefore, if the supplier chooses a contract $(t, q)$, the retailer will generate a profit $v(q) - t$. The retailer now needs to design a menu of contract to maximize her expected profit.

(a) (5 points) Formulate the retailer's contract design problem for finding the second best menu.

(b) (10 points) Solve for the second best menu.

(c) (5 points) Demonstrate "monotonicity", "efficiency at top", and "no rent at bottom". For $\theta_H$ and $\theta_L$, which is "top" and which is "bottom"?

3. (10 points) Read the article about Orbitz.com on the Wall Street Journal we mentioned in class. Write a summary of the story in your own words. Then write down your comments and thoughts. Besides of the method mentioned in the article, how would you identify consumers' willingness-to-pay?