IM 7011: Information Economics

Lecture 12: Moral Hazard
Chen and Huang (2013)

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November 25, 2013

Road map

- **Introduction**.
- Simplified model.
- Analysis.
- Original model and analysis.
- Extensions and conclusions.

Pricing data services

- We use **data services** everyday.
  - Text messages.
  - Dial-up or ADSL.
  - 3G/4G.
- How do sellers (e.g., ISPs) **price** these services?
  - Text messages: by **quantity**.
  - Dial-up: by **time**.
  - ADSL: by **bandwidth**.
  - 3G/4G: by volume (i.e., quantity).
- Why different data services are priced by different **pricing metrics**?
  - There are certainly **supply-side** reasons, e.g., technology limits.
  - Is there any **consumer-side** reasons?
- Practitioners often make (effective or ineffective) decisions without using **scientific** methods.
  - We want to know whether pricing metrics are chosen in a "good" way.

Pricing metrics

- Suppose a monopoly data service provider (**seller**) intends to provide the services to **consumers**.
  - In the basic model, the cost for offering services are omitted.
  - The seller wants to find the **revenue-maximizing pricing** plan.
- Consumers are **heterogeneous** on their willingness-to-pay for data usage and connection speed.
- As consumer types are hidden, the seller can only adopt second- or third-degree price discrimination.[1]
- We will focus on second-degree price discrimination with the following three **pricing metrics**:
  - Pricing by time (e.g., minutes).
  - Pricing by bandwidth (e.g., Mbps).
  - Pricing by quantity (e.g., Gigs).
- Which pricing metric is the best?

[1] Pricing by usage/choice or attribute/identity.

After-sales selections

- Consumers do not just have hidden types.
- They also have hidden (uncontrolled) **after-sales selections**.
  - When I am priced by time, I select connection speed (by selecting software/applications).
  - When I am priced by bandwidth, I select my time usage.
  - When I am priced by quantity, I select time or speed.
- Each consumer acts to maximize his own utility.
- The selection of pricing metrics must consider:
  - The heterogeneity of consumers (**hidden information**).
  - The after-sales selections (**hidden action**).

Research questions

- The seller wants to find the **revenue-maximizing pricing metric**.
  - By time, bandwidth, or quantity?
- To answer this question, she must be able to find the optimal (second-best) menu under each pricing metric.
  - Given each pricing metric, the seller solves a **nonlinear pricing problem** through contract design.
  - Multi-tiered pricing, unlimited usage pricing, or both?
- To solve the nonlinear pricing problem, the seller must be able to **anticipate** each consumers' after-sales selection.
- As researchers, we want to find the **driving forces** for a pricing metric to be revenue-maximizing.
  - When one is better than the other, and **why**?

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Pricing metrics

- A monopoly risk-neutral seller is facing three options:
  - Pricing by minutes (M).
  - Pricing by bandwidth (B).
  - Pricing by quantity ($Q \equiv BM$).
- For pricing by M and Q, we exclude fixed-up-to plans.
  - Fixed-up-to plans may arise as a consequence of optimization.
  - We do not specifically focus on such a restriction.
- Given a pricing metric, the seller designs a **price schedule**.
  - For example, under pricing by minutes, the seller designs a function $P^M(M)$ to translate a time usage M to a payment $P^M(M)$.
- A price schedule can be implemented as a **menu of contracts**.
  - For example, $P^M(\cdot)$ can be implemented as $\{(M(\theta), P^M(\theta))\}$, where θ is the consumer's type (to be detailed later).
  - A price schedule is an indirect mechanism; a menu is a direct one.

Consumers' utility function

- Let $\theta \sim \text{Uni}(0,1)$ be the consumers' type.
- In the **simplified** model,[2] the type-θ consumer's utility is[3]

$$u(B, M, \theta) = \begin{cases} \theta BM - \frac{1}{2}(BM)^2 & + \theta B - \frac{1}{2}B^2 & \text{if } BM \le \theta \text{ and } B \le \theta \\ \frac{1}{2}\theta^2 & + \theta B - \frac{1}{2}B^2 & \text{if } BM > \theta \text{ and } B \le \theta \\ \theta BM - \frac{1}{2}(BM)^2 & + \frac{1}{2}\theta^2 & \text{if } BM \le \theta \text{ and } B > \theta \\ \frac{1}{2}\theta^2 & + \frac{1}{2}\theta^2 & \text{if } BM > \theta \text{ and } B > \theta \end{cases}.$$

  - The first part ($\theta BM - \frac{1}{2}(BM)^2$ and $\frac{1}{2}\theta^2$) makes $u(\cdot)$ **increasing and concave** in Q.
  - They also make $u(\cdot)$ increasing and concave in M when B is fixed.
  - The second part ($\theta B - \frac{1}{2}B^2$ and $\frac{1}{2}\theta^2$) makes $u(\cdot)$ increasing and concave in B when Q is fixed.
  - **Unlimited usage** does not give unlimited utility.

[2] We remove some parameters from the paper's original model at this moment.
[3] The "if" condition in the paper should be a typo. The sign should be reversed.

More about consumers' utility function

- The functional form

$$\theta BM - \frac{1}{2}(BM)^2 + \theta B - \frac{1}{2}B^2$$

  has its limitations.
  - Consumers who have stronger preference for Q **also** have stronger preference for B.
  - Nevertheless, multi-dimensional screening is too hard.
- A higher time usage results in a higher utility **only if** it corresponds to a higher data usage.
  - Consuming more time itself does not make one happier.
- As there is no cost for offering the service, the **socially efficient** consumption maximizes each consumer's utility.
  - The FOC gives $B = \frac{\theta(1+M)}{1+M^2}$ and $M = \frac{\theta}{B}$, which imply $B = \theta$ and $M = 1$.
  - Will there be efficiency loss?

Timing

- The seller determines the pricing metric.
- The seller announces a pricing menu.
  - For example, if she prices by minutes, she announces $\{(M(\theta), P^M(\theta))\}$.
- Each consumer self-selects one contract in the menu.
- Each consumer adjusts the variable not specified in the contract.
  - For example, if the seller prices by minutes, the consumer chooses his own connection speed.

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- Introduction.
- Simplified model.
- **Analysis**.
  - Pricing by minutes.
  - Pricing by bandwidth.
  - Pricing by quantity.
  - Comparisons.
- Original model and analysis.
- Extensions and conclusions.

Pricing by minutes: after-sales selection

- Suppose the type-θ consumer has chosen $(M(\hat{\theta}), P^M(\hat{\theta}))$ in stage 3.
- In stage 4, he determines the bandwidth B to maximize his **net utility**

$$U^M(B|\theta,\hat{\theta}) = \theta B M(\hat{\theta}) - \frac{1}{2}\Big(BM(\hat{\theta})\Big)^2 + \theta B - \frac{1}{2}B^2 - P^M(\hat{\theta}).$$

- To maximize his net utility, the consumer chooses the bandwidth

$$B^*(\theta,\hat{\theta}) = \theta\left[\frac{1+M(\hat{\theta})}{1+M(\hat{\theta})^2}\right].$$

- The **effective utility** of choosing $(M(\hat{\theta}), P^M(\hat{\theta}))$ is

$$U^M(\theta,\hat{\theta}) = \frac{\theta^2}{2}\frac{\left[1+M(\hat{\theta})\right]^2}{1+M(\hat{\theta})^2} - P^M(\hat{\theta}).$$

- Let $U^M(\theta) \equiv \max\Big\{U^M(\theta,\theta), 0\Big\} \equiv \Big[U^M(\theta,\theta)\Big]^+$.

Pricing by minutes: contract design

- In stage 2, the seller solves

$$\begin{aligned}\Pi^M = \max_{M(\cdot),P^M(\cdot)} \quad & \mathbb{E}\Big[P^M(\theta)\Big] \\ \text{s.t.} \quad & U^M(\theta) \geq U^M(\theta,\hat{\theta}) \quad \forall \theta,\hat{\theta} \\ & U^M(\theta) \geq 0 \quad \forall \theta.\end{aligned}$$

- To solve this problem, we apply the standard technique for continuous-type problems and other recent results.

Pricing by minutes: optimal menu

- It turns out that a **fixed-fee** pricing plan is optimal.

Lemma 1

Under pricing by minutes, the optimal pricing plan is to charge a single fixed fee $P^M = \frac{4}{9}$ for an unlimited usage. The seller's expected revenue is $\Pi^M = \frac{4}{27}$.

  - By buying the unlimited time usage, the type-θ consumer's net utility becomes
  $$\frac{1}{2}\theta^2 + \frac{1}{2}\theta^2 - P^M.$$
  Therefore, he buys the service if and only if $\theta \geq \sqrt{P^M}$.
  - The seller then maximizes the expected revenue $P^M(1 - \sqrt{P^M})$.
- Price discrimination is **suboptimal**.
- In equilibrium the seller **does not screen** consumers!

Pricing by bandwidth: after-sales selection

- Suppose the type-θ consumer has chosen $(B(\hat{\theta}), P^B(\hat{\theta}))$ in stage 3.
- In stage 4, he determines the time usage M to maximize

$$U^B(M|\theta, \hat{\theta}) = \theta B(\hat{\theta})M - \frac{1}{2}\Big[B(\hat{\theta})M\Big]^2 + B(\hat{\theta})\theta - \frac{1}{2}B(\hat{\theta})^2 - P^B(\hat{\theta}).$$

  - M only appears in the first part (quantity).
- The consumer chooses the time usage $M^*(\theta, \hat{\theta}) = \frac{\theta}{B(\hat{\theta})}$.
- The effective utility of choosing $(B(\hat{\theta}), P^B(\hat{\theta}))$ is

$$U^B(\theta, \hat{\theta}) = \frac{1}{2}\theta^2 + B(\hat{\theta})\theta - \frac{1}{2}B(\hat{\theta})^2 - P^B(\hat{\theta}).$$

- Let $U^B(\theta) \equiv \max\Big\{U^B(\theta, \theta), 0\Big\} \equiv \Big[U^B(\theta, \theta)\Big]^+$.

Pricing by bandwidth: contract design

- In stage 2, the seller solves

$$\begin{aligned} \Pi^B = \max_{B(\cdot), P^B(\cdot)} \quad & \mathbb{E}\Big[P^B(\theta)\Big] \\ \text{s.t.} \quad & U^B(\theta) \geq U^B(\theta, \hat{\theta}) \quad \forall \theta, \hat{\theta} \\ & U^B(\theta) \geq 0 \quad \forall \theta. \end{aligned}$$

Pricing by bandwidth: optimal menu

- Now **multi-tiered** (usage-based) pricing is optimal.

Lemma 2

Under pricing by bandwidth, the optimal pricing plan satisfies

$$B^*(\theta) = 2\theta - 1 \quad \textit{and} \quad P^B(\theta) = 2\theta - \theta^2 - \frac{1}{2} + \frac{\underline{\theta}(2\underline{\theta}^2 - \underline{\theta} + 3)}{2(3\underline{\theta} - 2)}$$

for $\theta \geq \underline{\theta}$ *and* $B^*(\theta) = P^B(\theta) = 0$ *for* $\theta < \underline{\theta}$, *where* $\underline{\theta} = \frac{3+\sqrt{2}}{7}$ *is the lowest type of consumer that is served. The seller's expected revenue is* $\Pi^B = \frac{1}{6} - \underline{\theta}^2(\frac{3}{2} - \frac{7}{3}\underline{\theta})$.

  - Monotonicity: $B^*(\theta)$ is nondecreasing. Also no rent at bottom.
  - Efficiency at top: $B^*(\theta) = 2\theta - 1 = \theta \Leftrightarrow \theta = 1$.
- Price discrimination is optimal but some consumers should be ignored.
- **Quantity discount**: $B^*(\theta)$ is linear while $P^B(\theta)$ is strictly concave.

Pricing by quantity: after-sales selection

- Suppose the type-θ consumer has chosen $(Q(\hat{\theta}), P^Q(\hat{\theta}))$ in stage 3.
- In stage 4, he determines the bandwidth B to maximize[4]

$$U^Q(B|\theta, \hat{\theta}) = \theta Q(\hat{\theta}) - \frac{1}{2}Q(\hat{\theta})^2 + B\theta - \frac{1}{2}B^2 - P^Q(\hat{\theta}).$$

  - B only appears in the second part (bandwidth).
- The consumer chooses the bandwidth $B^*(\theta, \hat{\theta}) = \theta$.
- The effective utility of choosing $(B(\hat{\theta}), P^B(\hat{\theta}))$ is

$$U^Q(\theta, \hat{\theta}) = Q(\hat{\theta})\theta - \frac{1}{2}Q(\hat{\theta})^2 + \frac{1}{2}\theta^2 - P^Q(\hat{\theta}).$$

- Let $U^Q(\theta) \equiv \max\left\{U^Q(\theta, \theta), 0\right\} \equiv \left[U^Q(\theta, \theta)\right]^+$.

[4] As long as $Q(\hat{\theta}) = BM$, an equivalent result may be obtained by using the time usage M as the variable or by using both B and M as variables.

Pricing by quantity: contract design

- In stage 2, the seller solves

$$
\begin{aligned}
\Pi^Q = \max_{Q(\cdot), P^Q(\cdot)} \quad & \mathbb{E}\Big[P^Q(\theta)\Big] \\
\text{s.t.} \quad & U^Q(\theta) \geq U^Q(\theta, \hat{\theta}) \quad \forall \theta, \hat{\theta} \\
& U^Q(\theta) \geq 0 \quad \forall \theta.
\end{aligned}
$$

Pricing by quantity: optimal menu

- Again, multi-tiered (usage-based) pricing is optimal.

Lemma 3

Under pricing by quantity, the optimal pricing plan satisfies

$$Q^*(\theta) = 2\theta - 1 \quad \textit{and} \quad P^Q(\theta) = 2\theta - \theta^2 - \frac{1}{2} + \frac{\underline{\theta}(2\underline{\theta}^2 - \underline{\theta} + 3)}{2(3\underline{\theta} - 2)}$$

for $\theta \geq \underline{\theta}$ *and* $Q^*(\theta) = P^Q(\theta) = 0$ *for* $\theta < \underline{\theta}$, *where* $\underline{\theta} = \frac{3+\sqrt{2}}{7}$ *is the lowest type of consumer that is served. The seller's expected revenue is* $\Pi^Q = \frac{1}{6} - \underline{\theta}^2(\frac{3}{2} - \frac{7}{3}\underline{\theta})$.

- Identical to pricing by bandwidth!
- Consumers' effective utility is:
  - $\frac{1}{2}\theta^2 + B(\hat{\theta})\theta - \frac{1}{2}B(\hat{\theta})^2 - P^B(\hat{\theta})$ when pricing by bandwidth.
  - $Q(\hat{\theta})\theta - \frac{1}{2}Q(\hat{\theta})^2 + \frac{1}{2}\theta^2 - P^Q(\hat{\theta})$ when pricing by quantity.

Selection among pricing metrics

- Now we may find the revenue-maximizing pricing metric:

Proposition 1

- *A single contract is offered under pricing by minutes. A menu is offered under pricing by bandwidth or quantity.*
- *Because* $\Pi^M \approx 0.148 < 0.155 \approx \Pi^B = \Pi^Q$*, pricing by minutes is not revenue-maximizing.*
- *Because* $1 - \frac{2}{3} \approx 0.33 < 0.37 \approx 1 - \underline{\theta}$*, more consumers are served under pricing by bandwidth or quantity.*
- *Pricing by bandwidth and pricing by quantity are equivalent.*

- Pricing by minutes **cannot screen** consumers (with a fixed fee).
- Pricing by minutes is the least effective in **alleviating the moral hazard problem**.
  - Consumers are "too free": They can adjust bandwidth to affect both bandwidth and quantity.
  - In the other two cases, only one part can be adjusted.

Robustness of insights

- Are the insights robust?
  - Is pricing by minutes always inferior?
  - Are pricing by bandwidth and pricing by quantity always identical?
- To answer this question, a more general model is required.

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Original consumers' utility function

- In the original model in the paper, the type-θ consumer's utility function is

$$u(B,M,\theta)=\begin{cases}\delta\theta BM-\frac{1}{2\eta}(BM)^2 & +\ \theta B-\frac{1}{2\gamma}B^2 & \text{if } BM\leq\theta \text{ and } B\leq\theta\\ \frac{1}{2}\eta\delta^2\theta^2 & +\ \theta B-\frac{1}{2\gamma}B^2 & \text{if } BM>\theta \text{ and } B\leq\theta\\ \delta\theta BM-\frac{1}{2\eta}(BM)^2 & +\ \frac{1}{2}\gamma\theta^2 & \text{if } BM\leq\theta \text{ and } B>\theta\\ \frac{1}{2}\eta\delta^2\theta^2 & +\ \frac{1}{2}\gamma\theta^2 & \text{if } BM>\theta \text{ and } B>\theta\end{cases}.$$

  - $\delta > 1$ ($\delta < 1$): One is more (less) **sensitive** to changes in Q than B.
  - η (γ) increases: The marginal benefit of quantity (bandwidth) diminishes in a **slower** rate.

- With the more general utility function, do the results change?

More general insights

- The old results can now be generalized:

Proposition 2

- *A single contract is offered under pricing by minutes. A menu is offered under pricing by bandwidth or quantity.*
- *Because* $\Pi^M < \Pi^B$ *and* $\Pi^M < \Pi^Q$, *pricing by minutes is not revenue-maximizing.*
- *Pricing by bandwidth is revenue-maximizing if and only if* $\gamma \geq \delta^2 \eta$.

- Some insights are robust:
  - Pricing by minutes still cannot screen consumers.
  - Pricing by minutes is still suboptimal.
- Some are not:
  - Pricing by bandwidth and pricing by quantity are **not** identical.
  - **Both** of them may be revenue-maximizing.

Revenue maximization and moral hazard

- Why pricing by bandwidth is optimal if and only if $\gamma \geq \delta^2\eta$?
- It depends on which pricing metric is more effective in alleviating the **moral hazard** issue.
  - Under pricing by bandwidth, the utility is

$$\underbrace{\delta\theta B(\hat{\theta})M - \frac{1}{2\eta}\Big[B(\hat{\theta})M\Big]^2}_{\text{can be adjusted}} + B(\hat{\theta})\theta - \frac{1}{2\gamma}B(\hat{\theta})^2.$$

  - Under pricing by quantity, the utility is

$$\delta\theta Q(\hat{\theta}) - \frac{1}{2\eta}Q(\hat{\theta})^2 + \underbrace{B\theta - \frac{1}{2\gamma}B^2}_{\text{can be adjusted}}.$$

  - When γ is large, $B\theta - \frac{1}{2\gamma}B^2$ is large and pricing by quantity leaves the consumer **a too large room** for adjustment.
  - When δ or η is large, $\delta\theta B(\hat{\theta})M - \frac{1}{2\eta}[B(\hat{\theta})M]^2$ is large.

Revenue maximization and adverse selection

- Why pricing by bandwidth is optimal if and only if $\gamma \geq \delta^2\eta$?
- It also depends on which pricing metric is more effective in alleviating the **adverse selection** issue.
- For the functional form

$$\delta\theta BM - \frac{1}{2\eta}(BM)^2 + \theta B - \frac{1}{2\gamma}B^2 :$$

  - When $\delta < 1$, consumers are **more heterogeneous** in B than in Q.[5]
  - Pricing by bandwidth, which screens consumers according to their willingness-to-pay for B, is more effective.
  - When $\delta > 1$, consumers are more heterogeneous in Q than in B.
  - Pricing by quantity becomes more effective.

[5] In fact η and γ also have impacts on the heterogeneity. As the impacts are somewhat less apparent, we do not discuss them here.

ADSL vs. 3G/4G

- Does our theory apply to the current practices?
- Currently, few data services are priced by minutes.
  - Supply side: Controlling the quantity is more direct than controlling time usage.
  - Consumer side: Pricing by minutes is not revenue-maximizing.
- ADSL is typically priced by bandwidth.
  - ADSL consumers are **more heterogeneous** in applications they prefer (and thus in **bandwidth**).
  - Therefore, pricing by bandwidth is more effective.
- 3G/4G is typically priced by quantity.
  - Few 3G/4G consumers use speed-demanding applications. Most of them spend most of the time on simple browsing/searching. They are **less heterogeneous in bandwidth**.
  - Pricing by quantity is thus more effective.

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Extensions

- The model may be further extended in the following ways:
  - General utility functions: $U(B, M, \theta) = U^Q(Q, \theta) + U^B(B, \theta)$.
  - Bandwidth-insensitive utility functions: $U(B, M, \theta) = U(Q, \theta)$.
  - Aggregate bandwidth costs.
  - Disutility of waiting.
- In the presence of the last two supply-side issues:
  - Pricing by minutes is still suboptimal.
  - Pricing by bandwidth becomes relatively more attractive.

Conclusions

- Three pricing metrics for data services are studied.
  - Pricing by minutes, bandwidth, or quantity.
- Either pricing by bandwidth or pricing by quantity can be optimal.
  - Pricing by minutes is the worst in mitigating **information asymmetry**. The remaining moral hazard problem is the most significant.
  - Whether the seller should price by bandwidth or quantity also depends on the effectiveness of mitigating information asymmetry.
- Why is information asymmetry critical?
  - We want to earn revenues at the consumer side.
  - We do not know how consumers **like** our product.
  - We do not know how consumers will **use** our product.
- After-sales selections are also important when we design returns, warranties, and many other consumer-related policies.