Information Economics, Fall 2016
Problem Set 1

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1. Please answer the following questions.

   (a) Let $f(x_1, x_2) = 2x_1^5 + 3x_1^2 x_2 - x_2^3 + 3x_1$. Find the gradient $\nabla f(x_1, x_2)$ and Hessian $\nabla^2 f(x_1, x_2)$.

   (b) Let $f(x) = \ln(x^3 + 2x)e^{3x}$. Find $\frac{d}{dx} f(x)$.

   (c) Let $f(x) = x_1 x_2^2 + e^{2x_2} x_1$. Find $\int f(x) dx_2$ (you may ignore the constant).

   (d) Find $\frac{d}{dx} \int_0^x (t^3 + 3x - 2) dt$.

   (e) Let X be the outcome of rolling an unfair dice whose probability distribution is summarized in the following table:

x	1	2	3	4	5	6
$\Pr(X = x)$	0.2	0.3	0.1	0.1	0.1	0.2

   Find the expected value and variance of X.

   (f) Let $f(x) = kx^{1.5}$ be the probability density of a continuous random variable $X \in [0, 2]$. Find the value of k. Then find $\mathbb{E}[X]$.

   (g) Is $f(x) = x^{2.5} + 3x^2$ a convex function over $[0, \infty)$? Prove it mathematically.

   (h) Over what region is $g(x) = \ln x + 2x^2$ a strictly convex function? Prove it mathematically.

2. Consider the following nonlinear program

$$
\begin{aligned}
z^* = \max \quad & x_1 - x_2 \\
\text{s.t.} \quad & x_2 \geq -1 \\
& -x_1^2 - (x_2 + 2)^2 \leq -4.
\end{aligned}
$$

   (a) Draw the feasible region. Is it a convex set?

   (b) Graphically solve the problem.

   (c) Is there any local maximum that is not a global maximum? If so, find them.

   (d) Replace the second constraint by $-x_1^2 - (x_2 + 2)^2 \geq -4$. Redo Part (c).

3. Let F and G be two convex sets in $\mathbb{R}^n$. Prove or disprove that their intersection $F \cap G$ is also a convex set in $\mathbb{R}^n$.

4. Consider the monopoly pricing problem discussed in class. Suppose that now there is a competitor who sells the same product at price p_0. This competitor sticks to p_0 for no reason; it does not change the price no matter what happens. If a consumer wants to buy the product, she purchases the product from you only if your price is no greater than that from your competitor. In other words, if your price $p > p_0$, you will sell nothing for sure.

   (a) Formulate the seller's problem for maximizing its total expected profit. Show that it is a convex program.

   (b) Note that your program is a convex constrained program. For one-dimensional convex constrained program, the following strategy typically works: (1) find an unconstrained optimal solution, (2) if it is feasible, it is optimal, and (3) otherwise, find a boundary point that is closest to the unconstrained optimal solution. As you already know, the unique unconstrained optimal solution is $p^* = \frac{b+c}{2}$. As p^* may be greater than or less than p_0, apply the above strategy to analytically solve the seller's problem with this competitor who does not change its price.

(c) How does p^* change when a, b, or c changes? Provide economic intuitions to these mathematical results.

5. Consider the newsvendor problem discussed in class. Suppose that now unsold products can be sold to a recycling site at a price \$$d$ per unit. Obviously, we have $0 < d < c$.

   (a) Formulate the seller's problem of maximizing the expected profit.

   (b) Solve the problem and find the unique analytical optimal order quantity q^*.

   (c) How does q^* change when r, c, or d changes? Provide economic intuitions to these mathematical results.