Operations Research, Spring 2013
Project 1

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The problem

You were recently hired as a operations researcher in a big retailing company. While the company owns a lot of retail stores, it now wants to build several distributing centers (DC) to facilitate the replenishment process. There are many possible locations to build DCs on, and your job is to choose some locations to build DCs. Obviously, building a lot of DCs reduces the shipping costs from DCs to retail stores. Nevertheless, the construction costs may be too high. So you need to find a balance between shipping costs and construction costs.

You are given the following data. The existing retail stores are labeled as stores 1, 2, 3, ..., and m. Locations to build DCs are labeled as locations 1, 2, ..., and n. For store i and location j, the average shipping cost per truck is estimated to be d_{ij} dollars. For store i, the average number of truckloads it requires per year is estimated to be h_i. As an example, suppose we build a DC at location 1 and ship products to store 2 from location 1, then each year we need to pay $h_2 d_{21}$ dollars as the years shipping cost. For location j, building a DC there requires a construction cost. It has been determined that each DC can be used for twenty years. The one-time construction cost is assumed to be amortized linearly and the one-year construction cost is estimated to be f_j. As an example, if building a DC at location 1 costs 20,000,000 dollars, then $f_1 = 1,000,000$ dollars.

As an operations researcher, you decide to make a construction and replenishment plan. For each location, you need to determine whether a DC should be built. Moreover, you need to assign each store to exactly one DC, which means in the future products sold in this store will be shipped from that DC. Please note that while one DC can serve multiple stores, you cannot split the demand in one store to multiple DCs. Certainly you also cannot assign a store to a location at which no DC is built. Building DCs creates construction costs and assigning stores to DCs generates shipping costs. Your task is to find a feasible plan that minimizes the total costs.

1. Construct a linear integer program for the above task. All your decision variables should be binary.
2. Explain what may happen if you use continuous variables, whose values may be any real number between 0 and 1, rather than binary variables. If you defined different types of variables in Part 1, please explain this for each type respectively.
3. Try to design an algorithm to solve the program you formulate in Part 1. As you will learn in the course "Algorithms", the problem you formulated in Part 1 is an NP-hard problem (if your formulation is correct), which means most researchers in this world believe there is no algorithm that can find the optimal solution of this problem in a polynomial time. Therefore, what you should do is to design a polynomial-time algorithm (whose worst case running time, measured by the big-O notation, should be a polynomial function of n and m) that "usually" find a "near-optimal" solution. It will be even better if you can analyze when your algorithm performs well or badly.

Submission and grading

Please form a group of no more than five people for this project. Submit a hard copy of your report to the instructor's mail box at the first floor of the Management Building II by ***1:00pm, April 11, 2013***. The report cannot be longer than ten pages.

Your report will be graded based on the correctness (60%), organization and formatting (30%), creativity (10%), and the quality of oral presentation (bonus: 10%). All team members get the same grades. Among all teams that volunteer to present on April 15, 2013, at most five teams will be chosen based on the quality of the written report. The selection of teams for presentation will be announced shortly after the due date.