Operations Research, Spring 2014
Suggested Solution for Case assignmnet 3

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

(a) The main goal of this problem is to find the optimal number of kitchen sets that Furniture City can stock in its warehouse. There are eight kinds of features, total 30 items, and 20 kinds of kitchen sets. We define

$$x_i = \begin{cases} 1 & \text{if kitchen set } i \text{ can be supplied from the warehouse} \\ 0 & \text{otherwise} \end{cases}, i = 1, ..., 20$$

$$y_j = \begin{cases} 1 & \text{if item } j \text{ is selected to stock in the warehouse} \\ 0 & \text{otherwise} \end{cases}, j = 1, ..., 30$$

$$K_{ij} = \begin{cases} 1 & \text{if item } j \text{ is included in the kitchen set } i \\ 0 & \text{otherwise} \end{cases}, i = 1, ..., 20, j = 1, ..., 30$$

and x_i, y_j are decision variables and K_{ij} is a parameter. We also define $U = (2.5, 2.4, 2, 2, 3, 2)$ as the capacity of features. The item set in y_j, K_{ij} is the sequence order of tile, wallpaper, light fixure, cabinet, countertop, sink, dishwasher, and range. The BIP formulation is

$$\begin{aligned} \max \quad & \sum_{i=1}^{20} x_i \\ \text{s.t.} \quad & \sum_{j=4i-3}^{4i} y_j \le U_i \ \forall i = 1, ..., 6, \quad \sum_{j=25}^{30} y_j \le 4 \\ & y_j \ge x_i K_{ij} \quad \forall i = 1, ..., 20, j = 1, ..., 30 \\ & x_i, y_j \in \{0, 1\} \quad \forall i = 1, ..., 20, j = 1, ..., 30 \end{aligned}$$

The objective function maximizes the total selected kitchen sets. The first constraint ensures that the features satisfy theirs capacities. The third constraint ensures that when the kitchen set is selected, all the particular items in the set must be stocked in warehouse. After solving by AMPL, the solution is that x_4, x_{11}, x_{15}, and x_{20} are equal to one and the objective value is four. Those y_js that should be set to 1 are listed below.

Feature	Quantity	Sum	Capacity
y_2 (T2)	1	2	2.5
y_3 (T3)	1		
y_5 (W1)	1	2	2.4
y_7 (W3)	1		
y_9 (L1)	1	2	2
y_{11} (L3)	1		
y_{13} (C1)	1	2	2
y_{15} (C3)	1		
y_{17} (O1)	1	3	3
y_{18} (O2)	1		
y_{19} (O3)	1		
y_{21} (S1)	1	2	2
y_{23} (S3)	1		
y_{25} (D1)	1	4	4
y_{27} (R1)	1		
y_{29} (R3)	1		
y_{30} (R4)	1		

The remaining x and y not showed above are equal to zero. The solution here is not unique as there exist multiple combinations of kitchen sets that satisfy the constraints. E.g., $\{x_4, x_{13}, x_{14}, x_{16}\}$, and $\{x_4, x_{11}, x_{15}, x_{20}\}$. Howerver, the objective value are all four, Furniture City can at most stock four kitchen sets.

(b) We suggest that Furniture City should stock four kitchen sets x_4, x_{11}, x_{15}, and x_{20} in its warehouse. Therefore, the following items should stock in warehouse as well:

Feature	Items to stock
Floor Tile	20 square feet of Ivory textured tile and White checkered tile with blue trim
Wallpaper	five rolls of Plain ivory paper and Blue paper with marble texture
Light Fixtures	One unit of One large rectangular frosted fixture and One large oval frosted fixture
Cabinets	One unit of Light solid wood cabinets and Light wood cabinets with glass doors
Countertops	One unit of Plain light wood countertops, Stained light wood countertops, and White lacquer-coated countertops
Sinks	one unit of Sink with separate hot and cold water taps and Sink with one hot and cold water tap
Dishwashers	One unit of White energy-saving dishwasher
Ranges	One unit of White electric oven, White gas oven, and Ivory gas oven

(c) Owing to the additional space, we modify our model used in Part (a). We replace the constraint

$$\sum_{j=25}^{30} y_j \leq 4 \quad \text{by} \quad \sum_{j=27}^{30} y_j \leq 3.$$

After solving by AMPL, the solution is that x_4, x_8, x_{11}, x_{15}, and x_{20} are equal to one, the objective value is five, and those y_js that should be set to 1 are listed below.

Feature	Quantity	Sum	Capacity
y_2 (T2)	1	2	2.5
y_3 (T3)	1		
y_5 (W1)	1	2	2.4
y_7 (W3)	1		
y_9 (L1)	1	2	2
y_{11} (L3)	1		
y_{13} (C1)	1	2	2
y_{15} (C3)	1		
y_{17} (O1)	1	3	3
y_{18} (O2)	1		
y_{19} (O3)	1		
y_{21} (S1)	1	2	2
y_{23} (S3)	1		
y_{25} (D1)	1	2	2
y_{26} (D2)	1		
y_{27} (R1)	1	3	3
y_{29} (R3)	1		
y_{30} (R4)	1		

The remaining x and y not showed above are equal to zero. In Part (c), we suggest that Furniture City should stock one more kitchen set x_8 than Part (a), and stock one unit of Ivory energy-saving dishwasher in warehouse.

(d) In Part (d), we receive extra space and we need to modify our model used in Part (c). We replace the parameter U with $U = (2.5, 2.4, 3, 3, 4, 4)$. After solving by AMPL, the solution is that $x_4, x_8, x_{11}, x_{15}, x_{16}$, and x_{20} are equal to one, the objective value is six, and those y_js that should be set to 1 are listed below.

Feature	Quantity	Sum	Capacity
y_2 (T2)	1	2	2.5
y_3 (T3)	1		
y_5 (W1)	1	2	2.4
y_7 (W3)	1		
y_9 (L1)	1	3	3
y_{11} (L3)	1		
y_{12} (L4)	1		
y_{13} (C1)	1	3	3
y_{15} (C3)	1		
y_{16} (C4)	1		
y_{17} (O1)	1	4	4
y_{18} (O2)	1		
y_{19} (O3)	1		
y_{20} (O4)	1		
y_{21} (S1)	1	4	4
y_{22} (S2)	1		
y_{23} (S3)	1		
y_{24} (S4)	1		
y_{25} (D1)	1	2	2
y_{26} (D2)	1		
y_{27} (R1)	1	3	3
y_{29} (R3)	1		
y_{30} (R4)	1		

The remaining x and y not showed above are equal to zero. In Part (d), Furniture City can stock one more kitchen set x_{16} than Part (c). Therefore the following items should be added to stock in warehouse:

Feature	Items to stock
Light Fixtures	One unit of Three small frosted globe fixtures
Cabinets	One unit of Dark wood cabinets with glass doors
Countertops	One unit of Ivory lacquer-coated countertops
Sinks	One unit of Divided sink with separate hot and cold water taps and garbage disposal and Divided sink with one hot and cold water tap and garbage disposal