Operations Research, Spring 2014
Homework 4

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Note. The deadline of this homework is ***1pm, March 20, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (20 points; 10 points each) Consider the following LP

$$\begin{aligned} z^* = \min \quad & 4x_1 + 4x_2 + x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \leq 2 \\ & x_1 + 2x_2 \leq 3 \\ & x_1 + 2x_2 + 3x_3 \geq 3 \\ & x_i \geq 0 \quad \forall\, i = 1, ..., 3. \end{aligned}$$

   (a) Solve the Phase-I LP for an initial bfs to the standard form of the original LP.

   (b) Solve the Phase-II LP for an optimal solution to the original LP.

2. (15 points; 5 points each) Consider a simplex tableau for a minimization LP.

0	2	0	4	0	10
0	3	0	2	1	6
0	1	1	1	0	2
1	0	0	1	0	3

   (a) By using the smallest index rule, keep iterating starting from the tableau until the LP is solved. Write down all the tableaus and an optimal bfs or conclude that the LP is unbounded.

   (b) Is the LP degenerate? Why?

   (c) Is there any iteration having no improvement? If yes, indicate it. If no, simply answer "no".

3. (25 points) Suppose we have obtained the tableau

c_1	c_2	0	0	0	0	10
4	a_1	1	0	a_2	0	b
-1	-5	0	1	-1	0	2
a_3	-3	0	0	-4	1	3

   for a maximization problem.

   (a) (3 points) When is the tableau optimal?

   (b) (3 points) What condition ensures that the LP is unbounded?

   (c) (4 points) When is the current basic feasible solution degenerate?

   (d) (5 points) What condition ensures that the current solution can be improved by replacing x_6 as a basic variable with x_1?

   (e) (5 points) Suppose $b > 0$, when is the tableau optimal with other optimal solutions?

   (f) (5 points) Suppose $b = 0$, when is the tableau optimal with other optimal solutions?

4. (10 points; 5 points each) Consider a mathematical program

$$\begin{aligned} \min \quad & \max\{x_1, x_2\} \\ \text{s.t.} \quad & g_i(x_1, x_2) \leq 0 \quad \forall i = 1, ..., m. \end{aligned} \tag{1}$$

(a) Explain why the program in (1) is equivalent to

$$\begin{aligned} \min \quad & w \\ \text{s.t.} \quad & w \geq \max\{x_1, x_2\} \\ & g_i(x_1, x_2) \leq 0 \quad \forall i = 1, ..., m. \end{aligned} \tag{2}$$

Hint. At any optimal solution, one constraint will always be binding.

(b) Explain why the program in (3) is equivalent to

$$\begin{aligned} \min \quad & w \\ \text{s.t.} \quad & w \geq x_1 \\ & w \geq x_2 \\ & g_i(x_1, x_2) \leq 0 \quad \forall i = 1, ..., m. \end{aligned} \tag{3}$$

Note. Please note that the above equivalence holds no matter what other constraints we have (e.g., some of those g_is may be nonlinear). Please also note that the same technique applies for a maximum function with more than two arguments. Finally, please note that the same technique applies when we maximize a minimum function.

5. (20 points) IM county is trying to determine where to place one county fire station. The locations of the county's five major cities are given in the following coordinates measured in kilometers. City 1 is at $(20, 20)$; city 2 is at $(60, 10)$; city 3 is at $(40, 30)$; city 4 is at $(40, 60)$; city 5 is at $(30, 0)$. City 1 has in average 20 fires per year. For cities 2, 3, 4, and 5, the average numbers of fires are 30, 50, 20, and 25, respectively. The county wants to build the fire station in a location that minimizes the average "distance to travel". Because in the county roads all run in either an east-west or a north-south direction, we use Manhattan distances rather than Euclidean distances. For example, if the fire station were located at $(50, 40)$ and a fire occurred at city 4, the "distance to travel" is

$$|50 - 40| + |40 - 60| = 30$$

kilometers to the fire. Formulate an LP that can determine where the fire station should be located.

Hint. An absolute value function is nonlinear. However, it is a maximum function. The idea you learned in the previous problem helps!

Note. In a "linear" program, no integer variable is allowed!

6. (10 points) Try your best to read the textbook and then write down "I have tried my best to read the textbook in the past seven days."[1]

[1] A list of suggested sections to read can be founded in slides.