Operations Research, Spring 2014
Homework 5

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Note. The deadline of this homework is ***1pm, April 10, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (15 points) Apply the branch-and-bound algorithm to solve

$$\begin{array}{rrcrcr} \max & 3x_1 & + & 2x_2 & & \\ \text{s.t.} & 2x_1 & + & 5x_2 & \leq & 20 \\ & 2x_1 & - & x_2 & \leq & 4 \\ & \multicolumn{5}{c}{x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2.} \end{array}$$

   You do not need to write down the solution process for each subproblem. However, you must draw the branching tree with essential information. When multiple variables are fractional, branch on the one with the smallest index. When multiple nodes are alive, branch the one with the highest objective value.

2. (15 points) Apply the branch-and-bound algorithm to solve[1]

$$\begin{array}{rrcrcrcrcrcrcr} \max & 2x_1 & + & 3x_2 & + & 4x_3 & + & x_4 & + & 3x_5 & - & 2x_6 & & \\ \text{s.t.} & 4x_1 & + & 5x_2 & + & 3x_3 & + & x_4 & + & 4x_5 & + & x_6 & \leq & 11 \\ & \multicolumn{13}{l}{x_i \in \{0, 1\} \quad \forall i = 1, ..., 6.} \end{array}$$

   You do not need to write down the solution process for each subproblem. However, you must draw the branching tree with essential information. When multiple variables are fractional, branch on the one with the smallest index. When multiple nodes are alive, branch the one with the highest objective value.

3. (10 points) Caption Wei is trying to choose seven out of nine volleyball players for a contest. Nine players have been rated (on a scale of 1 for poor to 5 for excellent) according to their spiking, blocking, receiving, and passing abilities. The positions that each player is allowed to play and the player's abilities are listed in the table below, where S, L, H, M, and O stand for setters, liberals, outside hitters, middle hitters, and opposite.

Player	Position	Spiking	Blocking	Receiving	Passing
1	S	2	1	3	5
2	S or L	2	3	4	3
3	S or H or O	4	4	4	3
4	H or L	3	3	5	3
5	H or M	5	4	3	3
6	H or M or O	3	3	3	2
7	M	4	5	1	1
8	H or L	4	2	3	2
9	L	1	1	5	3

   The seven-player team must satisfy the following restrictions:

   - There must be at least two players who may play H, at least two players who may play M, at least one player who may play S, at least one player who may play O, and at least one player who may play L.

[1] The objective coefficient of x_6 is indeed negative.

- The average spiking level of the seven players must be at least 3.
- If player 1 is chosen, then player 3 cannot be chosen.
- If player 2 is chosen, then players 4 and 5 must both be chosen.
- Either player 6 or player 7 must start.
- At least two players who may play S must be chosen.

Given these constraints, Wei wants to maximize the total receiving ability of the starting team. Formulate an IP that will help him form his team.[2]

4. (10 points) A manufacturer can sell product 1 at a price of \$5 per unit and product 2 at a price of \$7 per unit. Nine units of raw material are needed to manufacture one unit of product 1, and seven units of raw material are needed to manufacture one unit of product 2. A total of 120 units of raw material are available. The setup costs for producing products 1 and 2 are \$30 and \$40, respectively. However, if both products are produced for positive amounts, there is a saving of \$20 in the setup cost (i.e., in total \$50 are paid). Formulate an IP that maximizes the profit.

5. (10 points) Given an undirected graph $G = (V, E)$, where V is the set of nodes and E is the set of edges, we want to select as few nodes as possible so that all nodes are either selected or adjacent to at least one selected node. Two nodes are said to be adjacent if they are connected by one edge. As an example, for graph

$$G_1 = (V_1, E_1) = \Big(\big\{1,2,3\big\}, \big\{[1,2],[2,3]\big\}\Big),$$

an optimal selection is to select just node 2 (to make nodes 1 and 3 adjacent to the selected node 2). As another example, for graph

$$G_2 = (V_2, E_2) = \Big(\big\{1,2,3,4\big\}, \big\{[1,2],[2,3]\big\}\Big),$$

an optimal selection is to select nodes 2 and 4. Now, for graph

$$\begin{aligned} G_3 = (V_3, E_3) = \Big(&\big\{1,2,3,4,5,6,7,8,9,10\big\}, \\ &\big\{[1,2],[1,5],[1,8],[2,3],[2,5],[2,6],[2,9],[3,4],[3,6],[3,10],[4,5],[4,7], \\ &[5,8],[5,10],[6,9],[7,8],[8,9],[9,10]\big\}\Big), \end{aligned}$$

formulate an IP that selects a minimum number of nodes so that all nodes are either selected or adjacent to a selected node.

6. (10 points) Given an undirected graph $G = (V, E)$, where V is the set of nodes and E is the set of edges, we want to select as few edges as possible so that every node is an endpoint of at least one selected edge. As an example, for graph

$$G_1 = (V_1, E_1) = \Big(\big\{1,2,3\big\}, \big\{[1,2],[2,3]\big\}\Big),$$

an optimal selection (which is also the only feasible one) is to select both edges. As another example, for graph

$$G_2 = (V_2, E_2) = \Big(\big\{1,2,3,4\big\}, \big\{[1,2],[2,3],[3,4]\big\}\Big),$$

[2] As always, for problems asking you to formulate a mathematical program, do not worry about whether the problem is feasible. If you know volleyball enough, you may find some of the restrictions not quite unreasonable. For simplicity, ignore them.

an optimal selection is to select edges $[1,2]$ and $[3,4]$. Now, for graph

$$\begin{aligned} G_3 = (V_3, E_3) = \Big(&\Big\{1,2,3,4,5,6,7,8,9,10\Big\}, \\ &\Big\{[1,2],[1,5],[1,8],[2,3],[2,5],[2,6],[2,9],[3,4],[3,6],[3,10],[4,5],[4,7], \\ &[5,8],[5,10],[6,9],[7,8],[8,9],[9,10]\Big\}\Big), \end{aligned}$$

formulate an IP that selects a minimum number of edges so that every node is an endpoint of at least one selected edge.

7. (30 points) In this problem, we would like to invite you to design problems for your classmates (again)! Please create a scenario in which one has a decision to make and the decision making can be supported by Integer Programming. Describe the situation clearly and formulate an LP that solves the decision making problem. There is no restriction on the situation: As long as the decision can be made based on the outcome of an IP, it is fine. Of course, a situation for which binary variables are required in the formulation is highly preferred. You may write either in English or Chinese. The expected complexity of your problem is that most of your classmates may formulate a correct LP in 15 to 20 minutes. It is not good if it is too easy or too hard. The breakdown of grades is: (1) 15 points for the preciseness of your problem description and correctness of your formulation AND solution, (2) 10 points for the complexity, and (3) 5 points for how interesting your problem is. Some best problems will be selected to become lecture or homework problems (subject to your approval). Provide your problem description, formulation, AND an optimal solution (generated by AMPL, for example).