Operations Research, Spring 2014
Suggested Solution for Homework 5

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. The branch-and-bound tree for solving this problem is depicted in Figure 1. The optimal solution is $(3, 2)$. The optimal objective value is 13.

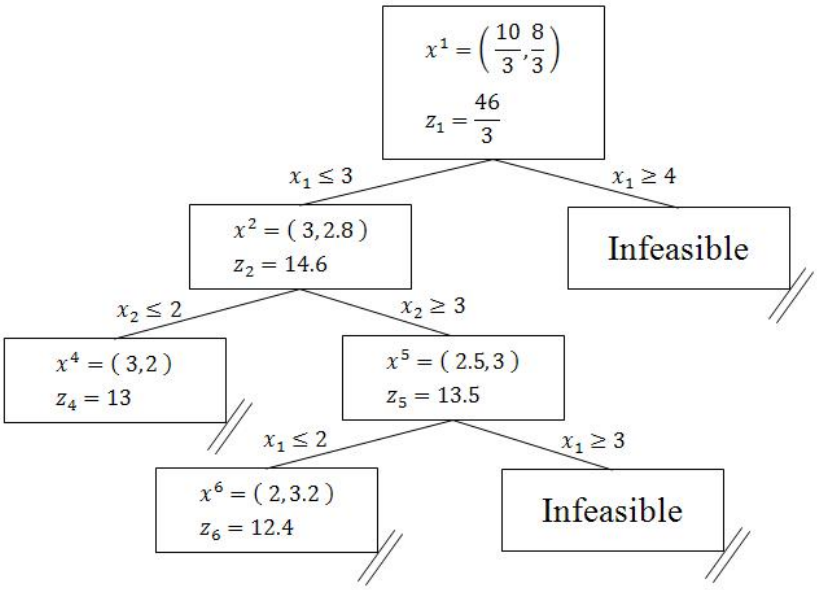

Figure 1: Branch-and-bound tree for Problem 1

2. The branch-and-bound tree for solving this problem is depicted in Figure 2. The optimal solution is $(1, 0, 1, 0, 1, 0)$. The optimal objective value is 9.

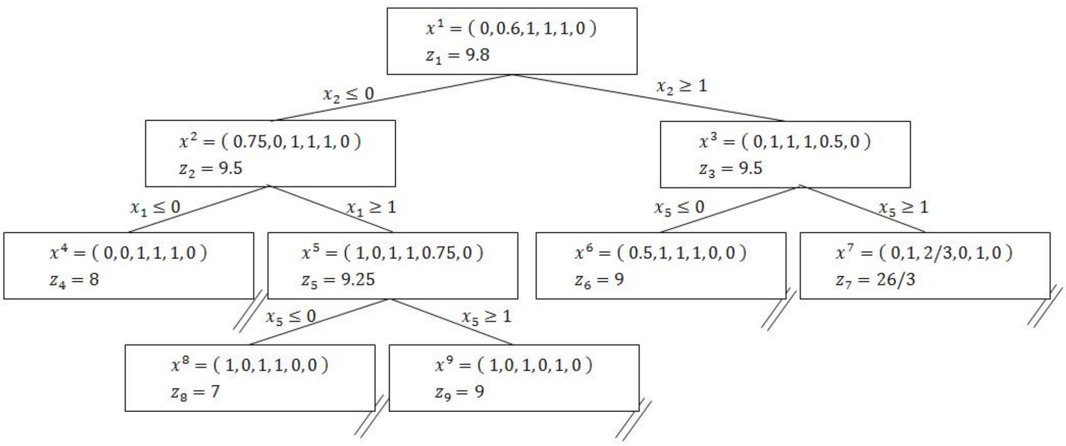

Figure 2: Branch-and-bound tree for Problem 2

3. We define

$$x_i = \begin{cases} 1 & \text{if player } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases}, i = 1, ..., 9$$

as our decision variables. We also define $R = (3, 4, 4, 5, 3, 3, 1, 3, 5)$ as the vector of receiving levels and $S = (2, 2, 4, 3, 5, 3, 4, 4, 1)$ as the vector of spiking levels. The complete formulation is

$$\begin{aligned} \max \quad & \sum_{i=1}^{9} R_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{9} x_i = 7 \\ & x_3 + x_4 + x_5 + x_6 + x_8 \geq 2 \\ & x_1 + x_2 + x_3 \geq 2 \\ & x_5 + x_6 + x_7 \geq 2 \\ & x_3 + x_6 \geq 1 \\ & x_2 + x_4 + x_8 + x_9 \geq 1 \\ & \sum_{i=1}^{9} S_i x_i \geq 21 \\ & x_1 + x_3 \leq 1 \\ & 2x_2 \leq x_4 + x_5 \\ & x_6 + x_7 \geq 1 \\ & x_i \in \{0, 1\} \quad \forall i = 1, ..., 9. \end{aligned}$$

The objective function maximizes the total receiving level. The first constraint chooses seven players. The second to sixth constraints ensure that we have enough players that can play setters, liberals, outside hitters, middle hitters, and opposite. The seventh constraint ensures that the average spiking level is at least 3. The eighth constraint ensures that if player 1 is chosen, then player 3 cannot be chosen." The nineth constraint ensures that if player 2 is chosen, then players 4 and 5 must both be chosen The tenth constraint ensures that either player 6 or player 7 must be chosen.

4. We define

$$\begin{aligned} x_i &= \text{ unit of product } i \text{ that will be produced, } i = 1,2 \\ y_i &= \begin{cases} 1 & \text{if manufacturer sets up for product } i \\ 0 & \text{otherwise} \end{cases}, i = 1, 2 \\ y_3 &= \begin{cases} 1 & \text{if manufacturer sells both product} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

as our decision variables. The IP formulation is

$$\begin{aligned} \max \quad & 5x_1 + 7x_2 - 30y_1 - 40y_2 + 20y_3 \\ \text{s.t.} \quad & 9x_1 + 7x_2 \leq 120 \\ & x_1 \leq \frac{120}{9} y_1 \\ & x_2 \leq \frac{120}{7} y_2 \\ & y_1 + y_2 \geq 2y_3 \\ & x_i \geq 0 \quad \forall i = 1, 2 \\ & y_i \in \{0, 1\} \quad \forall i = 1, 2, 3. \end{aligned}$$

The objective function maximizes the total profit. The first constraint ensures raw material are avaliable. The second and third constraints ensures that the product will be produced only when manufacturer sets up it. The fourth constraint ensures that the saving of setup cost occurs when both products are set up.

5. We define

$$
\begin{aligned}
S = \{&[1,2],[1,5],[1,8],[2,3],[2,5],[2,6],[2,9],[3,4],[3,6],\\
&[3,10],[4,5],[4,7],[5,8],[5,10],[6,9],[7,8],[8,9],[9,10]\}
\end{aligned}
$$

$$
x_i = \begin{cases} 1 & \text{if node } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases}, i = 1, ..., 10
$$

S is a parameter, and x_i are decision variables. The IP formulation is

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{10} x_i \\
\text{s.t.} \quad & x_i + x_j \geq 1 \quad \forall [i,j] \in S \\
& x_i \in \{0,1\} \quad \forall i = 1, ..., 10.
\end{aligned}
$$

The objective function minimizes the total selected nodes. The first constraint ensures that all nodes either be selected or its adjacent to a selected node.

6. We define

$$
\begin{aligned}
S = \{&[1,2],[1,5],[1,8],[2,3],[2,5],[2,6],[2,9],[3,4],[3,6],\\
&[3,10],[4,5],[4,7],[5,8],[5,10],[6,9],[7,8],[8,9],[9,10]\}
\end{aligned}
$$

$$
x_{ij} = \begin{cases} 1 & \text{if edge } [i,j] \text{ is selected} \\ 0 & \text{otherwise} \end{cases}, [i,j] \in S
$$

S is a parameter, and x_{ij} are decision variables. The IP formulation is

$$
\begin{aligned}
\min \quad & \sum_{[i,j] \in S} x_{ij} \\
\text{s.t.} \quad & \sum_{[i,j] \in S} x_{ij} + \sum_{[j,k] \in S} x_{jk} \geq 1 \quad \forall j = 1, ..., 10 \\
& x_{ij} \in \{0,1\} \quad \forall [i,j] \in S
\end{aligned}
$$

The objective function minimizes the total selected edges. The first constraint ensures that every node is an endpoint of at least one selected edge.

7. Omitted.