Operations Research, Spring 2014
Homework 9

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Note 1. The deadline of this homework is ***1pm, 22 May, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

Note 2. There are 20 bonus points for this Homework. You may earn at most 120 points!

1. (20 points; 5 points each) Consider the following two-player static game

	L	C	R
T	3, 4	2, 1	1, 0
M	2, 1	4, 2	2, 1
B	0, 4	2, 3	3, 2

   Let player 1 be the row player and player 2 be the column player.

   (a) Write down player 1's best response function: Given each action chosen by player 2, what is player 1's optimal action?

   (b) Write down player 2's best response function.

   (c) Does any player have any strategy that is strictly dominated by his another strategy?

   (d) Find all the Nash equilibria by finding all the intersections of the two best response functions.

2. (20 points; 5 points each) In a game, n players are asked to submit an integer between 1 and 100 simultaneously. The one who is closest to the half of the average of these n integers will win. Winners will share \$100 equally while losers get nothing. Each of them acts to maximize her payoff. Let $(x_1, x_2, ..., x_n)$ denote an action profile, where x_i is the integer chosen by player i.

   (a) Suppose $n = 2$. Is $(x_1, x_2) = (50, 50)$ a Nash equilibrium? Why or why not?

   (b) Suppose $n = 3$. Is $(x_1, x_2, x_3) = (33, 33, 33)$ a Nash equilibrium? Why or why not?

   (c) Suppose $n = 2$, find all the Nash equilibria, if any.

   (d) Suppose $n = 10$, find all the Nash equilibria, if any.

3. (20 points; 5 points each) Between two villages, there is a river. The two villages are discussing whether to invest to build a bridge. If the total amount invested achieves \$$T$, the bridge will be built and each village will save \$1000 for transportation annually. The bridge can be used for ten years and thus save the two villages in total \$20000. Each village has exactly one chance to choose an investment amount within \$0 and \$$T$. They will make the investment decision simultaneously. If the total investment is no lower than \$$T$, the remaining money will be equally shared by the two villages. Each village acts to maximize its own profit. Let (x_1, x_2) be an action profile, where x_i is the investment amount of village i.

   (a) Let $T = 8000$. Is (\$4000, \$4000) a Nash equilibrium? How about (\$0, \$0)? Why or why not?

   (b) Let $T = 8000$. Find all the Nash equilibria.

   (c) Let $T = 12000$. Is (\$6000, \$6000) a Nash equilibrium? How about (\$0, \$0)? Why or why not?

   (d) Let $T = 12000$. Find all the Nash equilibria.

4. (20 points; 5 points each) Consider the Cournot competition discussed in class.

(a) Suppose there are three firms and firm i chooses quantity q_i, $i = 1, 2, 3$. Again, the price will be $a - q$, where q is the aggregated quantity $q_1 + q_2 + q_3$. With a common production cost c, each firm acts to maximize its own profit. Find the unique Nash equilibrium.

(b) For n firms engaged in the Cournot competition, find the unique Nash equilibrium.

(c) Does the equilibrium quantity chosen by a single firm increase or decrease in n? Provide an economic intuition for your observation.

(d) Does the equilibrium profit earned by a single firm increase or decrease in n? Provide an economic intuition for your observation.

5. (20 points) For the Bertrand competition discussed in class, Show that integration benefits firms but hurt consumers.

6. (20 bonus points; 10 points each) Consider the Bertrand competition discussed in class. Suppose now the two firms are not identical: For firm i, $i = 1, 2$, the demand of its product is $q_i = a_i - p_i + bp_{3-i}$ and the unit production cost is c_i, where a_1, a_2, b, c_1, and c_2 are given parameters.

   (a) If $c_1 = c_2$ and $a_1 > a_2$, prove or disprove that in equilibrium firm 1 always chooses a higher price than firm 2.

   (b) If $c_1 > c_2$ and $a_1 = a_2$, prove or disprove that in equilibrium firm 1 always chooses a lower price than firm 2.