IM 2010: Operations Research, Spring 2014

Supply Chain Management

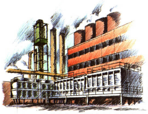
Ling-Chieh Kung

Department of Information Management
National Taiwan University

May 29, 20141

Supply chain management

- In operations research or management science, a subfield is called **supply chain management**.
  - A supply chain is a collection of firms such as suppliers, manufacturers, distributors, wholesalers, retailers, and salespeople that together deliver products to end consumers.



http://servagya.com

→ → →

http://www.hvsystems.co.uk

  - An extension of operations management (focusing on manufacturers).
- Strategic decisions: distribution channel structure, supplier selection, collaborative forecasting, etc.

Supply chain contracting

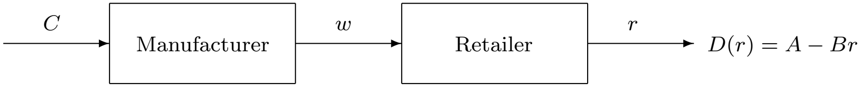
- Some firms operate its own supply chain.
- In most cases, a supply chain is **decentralized**.
  - Firms interact through **contracting**.
- Firms in a supply chain are teammates but also competitors.
  - A firm does not act for the chain's profit or other firms' profits.
  - A firm acts for its own profit.
- **Game theory** helps!
  - Key issues: **incentives** and **information**.
- A supply chain is also called a **distribution channel**.

Road map

- **Supply chain coordination**.
- Chain-to-chain competition.

Pricing in a supply chain

- Recall our supply chain pricing game:



- Suppose the supply chain is **decentralized**:
  - The retail price $r^* = \frac{BC+3A}{4B}$.
  - The retailer earns $\pi_{\text{R}}^* = \frac{(A-BC)^2}{16B}$.
  - The manufacturer earns $\pi_{\text{M}}^* = \frac{(A-BC)^2}{8B}$.
  - In total, they earn $\pi_{\text{C}}^* = \pi_{\text{R}}^* + \pi_{\text{M}}^* = \frac{3(A-BC)^2}{16B}$.
- Suppose the two firms **integrate**:
  - The optimal solution is $r^{\text{FB}} = \frac{BC+A}{2B} < r^*$.
  - In total, they earn $\pi_{\text{C}}^{\text{FB}} = \frac{(A-BC)^2}{4B} > \pi_{\text{C}}^*$.

Double marginalization

- **Decentralization** introduces inefficiency.
  - **Double marginalization**: The retail price is marked up **twice**.
  - The sales volume is smaller under decentralization.
  - the "total pie" becomes smaller.
- There is **incentive misalignment** in the supply chain.
- Inefficiency can be eliminated if the manufacturer chooses $w = C$.
  - This is impossible!
- Any solution?
  - Changing the game rules.
  - Using a different contract format.

Two-part tariffs

- A **two-part tariff** consists of a per-unit price w and a lump-sum fee t.
  - Buying q units requires $wq + t$ dollars.
- In this case, the retailer's behavior is identical.
  - The optimal retail price is still $r^{**}(w) = \frac{Bw+A}{2B}$. It earns $\frac{(A-Bw)^2}{4B} - t$.
- The manufacturer solves

$$\begin{aligned} \pi_{\mathrm{M}}^{**} = \max_{w \geq 0, t \geq 0} & \quad (w - C)\left(\frac{A - Bw}{2}\right) + t \\ \text{s.t.} & \quad \frac{(A - Bw)^2}{4B} - t \geq 0. \end{aligned} \tag{1}$$

Proposition 1

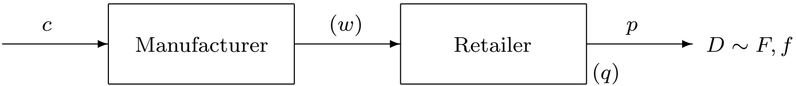
For the problem in (1), the optimal solution is $t^{**} = \frac{(A-BC)^2}{4B}$ *and* $w^{**} = C$*. The associated objective value is* $\pi_{\mathrm{M}}^{**} = \frac{(A-BC)^2}{4B}$.

Supply chain coordination

- A two-part tariff can **coordinate** the supply chain.
  - The equilibrium outcome is **(socially) efficient**.
  - The manufacturer provides enough **incentives** to induce the retailer to choose the efficient retail price.
- In equilibrium, the manufacturer takes all; the retailer gets nothing.
- But **win-win** can be achieved!
  - t may be adjusted to make the retailer profitable.
  - E.g., $t > \pi_R^* = \frac{(A-BC)^2}{16B}$ is attractive.

Indirect newsvendor

- How about the indirect newsvendor channel?



- They try to maximize:
  - The retailer: $\pi_{\text{R}}(q) = p\mathbb{E}[\min\{D, q\}] - wq$.
  - The manufacturer: $\pi_{\text{M}}(w) = (w - c)q^*$, where $q^* \in \text{argmax}_q\{\pi_{\text{R}}(q)\}$.
- If the supply chain is decentralized:
  - $w^* > c$ and $F(q^*) = 1 - \frac{w^*}{p}$.
- If the two firms integrate:
  - $F(q^{\text{FB}}) = 1 - \frac{c}{p}$; $q^* < q^{\text{FB}}$.
- Any contract to coordinate the supply chain?

Risk-sharing contracts

- The retailer orders too few because $w > c$.
  - Overage is too costly.
- The **risk** of overage is too high.
  - The retailer takes all the risk while the manufacturer is risk-free.
- A **risk-sharing** contract helps.
- In particular, a **return** (buy-back) contract works.
  - The retailer is allowed to return (all or some) unsold products to get (full or partial) credits.
- Contractual terms:
  - w is the wholesale price.
  - r is the **return credit** (buy-back price).
  - $(w, r) = (w, 0)$ reduces to the wholesale contract;
  - $(w, r) = (w, w)$ is a full return contract.

Expected profits

- Under a return contract (w, r), the retailer's expected profit is

$$\pi_{\mathrm{R}}(q) = \int_0^q \Big[xp + (q - x)r\Big] f(x)dx + \int_q^\infty qpf(x)dx.$$

- Let $q^* \in \operatorname{argmax}_{q \geq 0} \pi_{\mathrm{R}}(q)$. The manufacturer's expected profit is

$$\pi_{\mathrm{M}}(w, r) = q^*(w - c) - \int_0^{q^*} (q^* - x)rf(x)dx.$$

- The expected supply chain profit is

$$\pi_{\mathrm{C}}(q) = -cq + \int_0^q xpf(x)dx + \int_q^\infty qpf(x)dx.$$

Efficient inventory level

- From the supply chain's perspective, this is still the same problem.
- The efficient inventory level q^{FB} satisfies $F(q^{\text{FB}}) = 1 - \frac{c}{p}$.
- Questions:
  - Is there a contract (w, r) that induces the retailer to order q^{FB}?
  - Does that contract benefit both players (compared with the optimal wholesale contract)?

Retailer's ordering strategy

- Under a return contract, the retailer's expected profit is

$$\pi_{\text{R}}(q) = \int_0^q \Big[xp + (q-x)r\Big] f(x)dx + \int_q^\infty qpf(x)dx.$$

- We then have

$$\begin{aligned}\pi'_{\text{R}}(q) &= -w + \int_0^q rf(x)dx + \int_q^\infty pf(x)dx \\ &= -w + p - (p-r)F(q).\end{aligned}$$

and $\pi''_{\text{R}}(q) \leq 0$.

- To induce the retailer to order q^{FB}, we need $\pi'_{\text{R}}(q^{\text{FB}}) = 0$, i.e.,

$$\pi'_{\text{R}}(q^{\text{FB}}) = -w + p - (p-r)F(q^{\text{FB}}) = -w + p - \frac{(p-c)(p-r)}{p} = 0.$$

Coordinating return contracts

- Is there a coordinating return contract?

Proposition 2

- $\pi'_{\text{R}}(q^{FB}) = 0$ *if and only if* $w = p - \frac{(p-c)(p-r)}{p}$.
- *For any* p *and* c*, a pair of* $w \in [c, p]$ *and* $r \in [0, w]$ *exist to satisfy the above equation.*

Proof. The first part is immediate. According to the equation, we need $r = \frac{p(w-c)}{p-c}$. Then $w \leq p$ implies $r = \frac{p(w-c)}{p-c} \leq w$ and $c \leq w$ implies $r = \frac{p(w-c)}{p-c} \geq 0$. Such an r thus exists. □

- How about profit splitting?

Profit splitting

- Under a return contract, channel coordination requires

$$w = p - \frac{(p-c)(p-r)}{p} = c + \left(\frac{p-c}{p}\right)r.$$

  - When $w = c$, we need $r = 0$. In this case, $\pi^*_{\text{M}} = 0$ and $\pi^*_{\text{R}} = \pi^*_{\text{C}}$.
  - When $w = p$, we need $r = p$. In this case, $\pi^*_{\text{M}} = \pi^*_{\text{C}}$ and $\pi^*_{\text{R}} = 0$.
- And these functions are all **continuous**!
  - The supply chain expected profit may be split arbitrarily.
  - Win-win is possible.

Remarks

- For this problem, there are other coordinating contracts.
  - E.g., revenue-sharing contracts.
  - Key: incentives.
- In practice, the manufacturer may pay the retailer without asking for the physical goods.
- Two-part tariffs and return contracts may be actually **win-win-win**.
  - **Consumers** also benefit from supply chain coordination.
- In general, a coordinating contract is not always win-win.

Road map

- Supply chain coordination.
- **Chain-to-chain competition**.

Introduction

- In a distribution channel, the **channel structure** may be an issue.
  - In the previous two sections, the channel/supply chain structure cannot be altered: Integration is not an option of either firm.
  - Sometimes a firm needs to **decide** its channel structure.
- Should a manufacturer **downwards integrate** or not?
- Today let's introduce a nontrivial driving force discovered by a seminal work done by McGuire and Staelin (1983).[1]
  - It is a choice between integration and decentralization.
  - It is a choice between direct channel and indirect channel.
  - It is an application of **game theory**.

[1] McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(1) 115–130.

Research scope

- In practice, we see **exclusive** retail stores.
  - An exclusive retail store sells products only from **one** manufacturer.
  - It may be a **company store** or a **franchise store**.
- In what industries do we see them?
  - Gasoline, new automobiles, fast food restaurants, etc.
- What determines a manufacturer's decision?
  - Company stores or franchise stores?
- Under **competition**, the paper searches for conditions for the **industry equilibrium** to have a integrated channel (with a company store) or a decentralized channel (with a franchise store).

Model

- There are two manufacturers in a given region.
- They are selling different but **substitutable** products.
  - The demand of each product depends on both prices.
  - If both of them choose to sell through a company store, they play the **Bertrand game**.
- Each of them may independently decides whether to **delegate to a retailer** (insert one level into the channel).
  - In this case, the manufacturer sets a wholesale price and the retailer sets a retail price.
  - The two players in the channel play the **channel pricing** game.[2]
- Each of the manufacturer decides whether to **downwards integrate**.

[2] In previous lectures, we call this the supply chain pricing game.

Model

- There are three possible **industry structures**:
  - Pure integration (II: Integration–Integration).
  - Pure decentralization (DD: Decentralization–Decentralization).
  - Mixture (ID: Integration–Decentralization or DI).
- There are two manufacturers.
  - Each manufacturer has a downstream retail store (retailer).
  - The retail store is either a company store (under integration) or a franchise store (under decentralization).
- The demands at retail stores 1 and 2, respectively, are

$$q_1 = 1 - p_1 + \theta p_2 \text{ and}$$
$$q_2 = 1 - p_2 + \theta p_1.$$

  - The industry demand is normalized to 2 when both prices are zero.
  - $\theta \in [0, 1)$ measures the **substitutability** between the two products.

Pricing games

- Under II, manufacturer i sets retail price p_i to solve

$$\pi_i^{\mathrm{I}} \equiv \max_{p_i} \; p_i q_i, \quad i = 1, 2,$$

where π_i^{I} is the profit of channel i under integration.
- Under DD:
  - First manufacturer i sets wholesale price w_i to solve

$$\pi_i^{\mathrm{M}} \equiv \max_{w_i} \; w_i q_i, \quad i = 1, 2.$$

  - Then retailer i sets retail price p_i to solve

$$\pi_i^{\mathrm{R}} \equiv \max_{p_i} \; (p_i - w_i) q_i, \quad i = 1, 2.$$

  - π_i^{M} and π_i^{R} are the profits of the manufacturer and retailer in channel i under decentralization.

Pricing games

- Under ID:
  - First manufacturer 2 sets wholesale price w_2 to solve

$$\hat{\pi}_2^{\text{M}} \equiv \max_{w_2} \; w_2 q_2.$$

  - Then manufacturer 1 and retailer 2 set retail prices p_1 and p_2 to solve

$$\hat{\pi}_1^{\text{I}} \equiv \max_{p_1} \; p_1 q_1 \text{ and}$$
$$\hat{\pi}_2^{\text{R}} \equiv \max_{p_2} \; (p_2 - w_2) q_2.$$

- DI is similar to ID.
- We have dynamic games with **embedded static games**!
- To complete our analysis, we apply **backward induction**:
  - Given any industry structure, find the equilibrium prices and profits.
  - Find the equilibrium industry structures.

Illustrative analysis: the DD structure

- Suppose the two manufacturers have chosen to have franchise stores.
- Let $\pi_i^{\mathrm{R}}(p_i) = (p_i - w_i)q_i = (p_i - w_i)(1 - p_i + \theta p_{3-i})$, where w_is are announced by the manufacturers.
- The two retailers solve

$$\pi_i^{\mathrm{R}} \equiv \max_{p_i} \ \pi_i^{\mathrm{R}}(p_i), \quad i = 1, 2.$$

- If (p_1^*, p_2^*) is a Nash equilibrium, retailer i's price p_i^* satisfies

$$\frac{\partial}{\partial p_i} \pi_i^{\mathrm{R}}(p_i) \bigg|_{p_i = p_i^*} = 1 - 2p_i^* + \theta p_{3-i}^* + w_i = 0, \quad i = 1, 2.$$

- A unique Nash equilibrium satisfies

$$p_i^* = \frac{1}{2 - \theta} + \frac{2w_i + \theta w_{3-j}}{(2 + \theta)(2 - \theta)}, \quad i = 1, 2.$$

Intuitions behind the equilibrium retail prices

- Consider the equilibrium retail prices

$$p_i^* = \frac{1}{2-\theta} + \frac{2w_i + \theta w_{3-i}}{(2+\theta)(2-\theta)}, \quad i = 1, 2.$$

- Do they make sense?
  - p_i^* goes up when w_i goes up.
  - p_i^* goes up when w_{3-i} goes up.
  - w_i has a larger effect on p_i^* than w_{3-i} does.
  - When $\theta = 0$, does p_i^* degenerate to that in a channel pricing game?
- Given these prices, the equilibrium demands are

$$q_i^* = \frac{1}{2-\theta} - \frac{(2-\theta^2)w_i - \theta w_{3-i}}{(2+\theta)(2-\theta)}, \quad i = 1, 2.$$

  Do they make sense?
- Let's continue to the manufacturers' problems.

The manufacturers' problems

- Let $\pi_i^{\mathrm{M}}(w_i) = w_i q_i^* = w_i \left[\frac{1}{2-\theta} - \frac{(2-\theta^2)w_i - \theta w_{3-i}}{(2+\theta)(2-\theta)} \right]$, the manufacturers solve

$$\pi_i^{\mathrm{M}} \equiv \max_{w_i} \ \pi_i^{\mathrm{M}}(w_i), \quad i = 1, 2.$$

- If (w_1^*, w_2^*) is a Nash equilibrium, manufacturer i's price w_i^* satisfies

$$\frac{\partial}{\partial w_i} \pi_i^M(w_i) \bigg|_{w_i = w_i^*} = \frac{1}{2-\theta} - \frac{2(2-\theta^2)w_i^* - \theta w_{3-i}^*}{(2+\theta)(2-\theta)} = 0, \quad i = 1, 2.$$

- The equilibrium wholesale prices are

$$w_1^* = w_2^* = \frac{2+\theta}{4-\theta-2\theta^2}.$$

The complete equilibrium

- The equilibrium wholesale prices are $w_1^* = w_2^* = \frac{2+\theta}{4-\theta-2\theta^2}$.
- The equilibrium retail prices are

$$p_1^* = p_2^* = \frac{2(3-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)}.$$

- The equilibrium demands are

$$q_1^* = q_2^* = \frac{2-\theta^2}{(2-\theta)(4-\theta-2\theta^2)}.$$

- The manufacturers' equilibrium profits are

$$\pi_1^{\mathrm{M}} = \pi_2^{\mathrm{M}} = \frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}.$$

- The retailers' equilibrium profits and the equilibrium channel profits can also be found.

Other industry structures

- For other industry structures, i.e., ID, DI, and II, we may find all the equilibrium outcomes.
- In particular, the manufacturers' equilibrium profits (the channel profit under integration) can be found.
- The four pairs of the manufacturers' equilibrium profits is the basis for solving the **channel structure game**.
  - There are two players.
  - They make decisions simultaneously.
  - Each of them has two options: integration of decentralization.
  - The payoff matrix can be constructed by solving the four pricing games.

The channel structure game

- The payoff matrix:

M2

		I	D
M1	I	$\frac{1}{(2-\theta)^2}$ $\frac{1}{(2-\theta)^2}$	$\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$ $\left[\frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)}\right]^2$
	D	$\left[\frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)}\right]^2$ $\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$	$\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$ $\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$

- Is there any (pure-strategy) Nash equilibrium?

Equilibrium channel structures: polar cases

- Find all the Nash equilibria for the two polar cases:

M2

M1	I	D
I	$\frac{1}{4}, \frac{1}{4}$	$\frac{1}{4}, \frac{1}{8}$
D	$\frac{1}{8}, \frac{1}{4}$	$\frac{1}{8}, \frac{1}{8}$

$(\theta = 0)$

M2

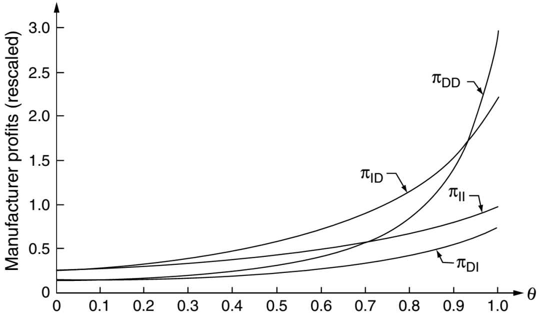
M1	I	D
I	$1, 1$	$\frac{9}{4}, \frac{3}{4}$
D	$\frac{3}{4}, \frac{9}{4}$	$3, 3$

$(\theta = 1)$

- DD is an **equilibrium** when $\theta = 1$!
- As all functions are continuous in $\theta \in [0, 1]$, DD must be an equilibrium for **large enough** θ.
- Let's do the complete analysis.

Equilibrium channel structures: general cases

Figure 2 Manufacturer's Profits as a Function of θ for Pure and Mixed Distribution Systems When Franchises Are Given Away



(McGuire and Staelin, 1983)

- $\pi_{\text{II}} > \pi_{\text{DI}}$: Mixture is never an equilibrium. II is always an equilibrium.
- If $\theta < 0.931$, $\pi_{\text{ID}} > \pi_{\text{DD}}$: DD is not an equilibrium. II is the only equilibrium.
- If $\theta > 0.931$, $\pi_{\text{DD}} > \pi_{\text{ID}}$: II is still an equilibrium. DD is **another equilibrium**.
- $\pi_{\text{DD}} > \pi_{\text{II}}$ if $\theta > 0.708$: **prisoners' dilemma** for $\theta \in (0.708, 0.931)$.

Incentives for decentralization

- Even though the retailer is not stronger than the manufacturer, a manufacturer may want do decentralization.
  - This happens when θ is high, i.e., the products are quite similar or the **competition is quite intense**.
- What is the incentive for the manufacturer to do so?
- According to the paper:

  Manufacturers in a duopoly are better off if they can **shield** *themselves from this environment by inserting privately-owned profit maximizers between themselves and the ultimate retail market.*

- "The competition is so intense that I'd better find someone to **fight for me**. I'd better not to engage in the competition directly."
- Is there an explanation from the perspective of efficiency?

Decentralization can be more efficient

- If the manufacturers are better off by doing pure decentralization, pure decentralization must generating a higher system profit.
- Why is DD more efficient than II?
- Suppose currently it is II.
  - The two manufacturers play the Bertrand game and consequently the equilibrium **prices are too low**.
- If they change to DD, each channel now has one additional layer of intermediary and the **price goes up**.
- Decentralization makes the prices **closer to the efficient level**.
- The pie becomes larger!

Decentralization provides credibility

- Under pure integration, the prices are too low and the two manufacturers are trapped in a prisoners' dilemma.
  - They know this. They know that together raising prices is win-win.
  - However, the promise to raise a price is **non-credible**.
  - They must somehow show that "I am (we are) forced to raise the price."
  - Having one additional layer provides **credibility**.
- Doing decentralization provides **incentives** for the competitor to raise her price (because she knows that I will raise my price).

Integration vs. decentralization

- Why integration fails? You told me integration is always optimal!
- The fact is **complete integration** is always optimal.
  - If the four firms are all integrated, the system is efficient.
  - But when complete integration is impossible (i.e., no manufacturer can horizontally integrate with the other), **partial integration** may be worse than **no integration** (i.e., decentralization).
- This is the so-called "Principle of the second best".
  - When you can control everything, do it.
  - When you cannot control everything, it may be better to control nothing.

Extensions and conclusions

- Extensions:
  - When the manufacturers act to maximize channel profits (probably with a coordinating contract, DD is an equilibrium if $\theta > 0.771$.[3]
  - When a manufacturer can set a sales quota or a price ceiling for its retailer, the result is still valid.
  - When the two manufacturers collude, they will downwards integrate.
  - The insight remains valid under other game structures or sequences.
- Conclusions:
  - A reason for a manufacturer to delegate to a retailer is provided.
  - A manufacturer may do so when the competition is intense.
    - Having one additional layer drives the originally too-low prices up.
  - The principal of the second best.
- If you are interested in this subject, take "Information Economics"!

[3] The region for DD to be an equilibrium is enlarged. Why?