Operations Research, Spring 2016
Homework 1

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1 Problems

1. (30 points; 15 points each) During the next six months IEDO company has the following demands for air conditioners at Taipei: month 1, 2500; month 2, 4000; month 3, 4500; month 4: 4200; month 5: 3800, month 6: 4400. The demands at Kaohsiung are: month 1, 3500; month 2, 4500; month 3, 4800; month 4: 5300; month 5: 4000, month 6: 4500. Air conditioners can be produced in either Hsinchu or Taoyuan. It takes 2 hours of skilled labor to produce an air conditioner in Hsinchu, and 2.5 hours in Taoyuan. It costs \$400 to produce an air conditioner in Hsinchu, and \$350 in Taoyuan. It costs \$20, \$30, \$50, and \$60 to ship an air conditioner from Taoyuan to Taipei, Hsinchu to Taipei, Hsinchu to Kaohsiung, and Taoyuan to Kaohsiung. During each month, 4000 hours of skilled labors are available at each of Taoyuan and Hsinchu. It costs \$80 to hold an air conditioner in inventory for a month. At the beginning of month 1, IEDO has 1000 air conditioners in stock at Hsinchu.

   (a) Suppose that IEDO must meet (on time) all the demands. Formulate an LP whose solution will tell IEDO how to minimize the cost of meeting air conditioner demands for the next six months.

   (b) Suppose that those demands are the maximum number that IEDO may sell, and IEDO can decide the sales quantity in each month. Each air conditioner can be sold at \$600. Formulate an LP whose solution maximizes the profit of selling air conditioners for the next six months.

2. (20 points; 5 points each) Consider the following LP

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + x_2 \leq 8 \\ & x_1 \text{ urs.}, x_2 \geq 0. \end{aligned}$$

   (a) Graphically solve the LP.

   (b) Find all the basic feasible solutions of the LP.

   (c) Use the simplex method with the smallest index rule to solve the LP. Write down all the derivations and then clearly indicate your conclusion.

   (d) Change the objective function to max $x_1 + 2x_2$. Use the simplex method with the smallest index rule to solve the LP.

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3. (10 points) Consider the following LP:

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 \leq 4 \\ & x_1 + 2x_2 + 3x_3 \leq 9 \\ & x_3 \geq 3 \\ & x_i \geq 0 \quad \forall i = 1, ..., 3. \end{aligned}$$

Use the simplex method with the smallest index rule to solve it.

4. (20 points; 5 point each) Consider the following LP:

$$\begin{aligned} \max \quad & x_1 + 3x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 3 \\ & -x_1 + 2x_2 \leq 8 \\ & 3x_1 + x_2 \leq 18 \\ & x_i \geq 0 \quad \forall i = 1, 2. \end{aligned}$$

   (a) Graphically solve the LP.

   (b) Use the simplex method to solve this LP. When there are multiple variables to enter, enter the one whose reduced cost has the highest magnitude (i.e., whose absolute value is the higest); if there are multiple variables having the same highest magnitude, use the smallest index rule. When there are multiple variables to leave, use the smallest index rule.

   (c) Use the simplex method to solve this LP. Whenever there is a tie, use the smallest index rule for tie-breaking.

   (d) Comment on whether the "highest-magnitude reduced cost" rule solves an LP faster than the smallest index rule.

5. (20 points; 5 points each) Consider the LP in Problem 4. Let the slack variables for constraints 1, 2, and 3 be x_3, x_4, and x_5, respectively. Consider the basis $B = (x_2, x_4, x_5)$.

   (a) (5 point) Find A_B, A_N, c_B, c_N, and b.

   (b) (5 points) Use the reduced cost $\bar{c}_N^T = c_B^T A_B^{-1} A_N - c_N^T$ to find an entering variable. Let that entering variable be x_j.

   (c) (5 points) Continue from Part (b). Use $A_B^{-1}b$ and $A_B^{-1}A_j$ to do the ratio test to find a leaving variable.

   (d) (5 points) Continue from Part (c). Use the results in Parts (b) and (c) to change the basis. Starting from the new basis, redo Parts (b) and (c) to go to the next basis.

2 Submission rules

The deadline of this homework is 2pm, March 21, 2016. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building 2 by the due time. Works submitted between 2pm and 3pm will get 10 points deducted as a penalty. Submissions later than 3pm will not be accepted. Each student must submit her/his individual work.