Operations Research, Spring 2016

Pre-lecture Problems for Lecture 13: Algorithms for Nonlinear Programming

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Note. The deadline of submitting the pre-lecture problem is ***10:10 am, May 19, 2015***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Let's solve $\min_{x \in \mathbb{R}^2} \; f(x) = x_1^2 + 2x_2^2$ by gradient descent. In each iteration, let the step size be that bringing you to the global minimum along the improving direction.

   (a) Find the gradient of $f(x)$.

   (b) Let $x^0 = (1, 0)$ be the initial solution. Run on iteration of gradient descent to find the next solution x^1.

   (c) Starting from x^1, run one more iteration of gradient descent to find the next solution x^2.

2. (0 point) Let's solve $\min_{x \in \mathbb{R}^2} \; f(x) = x_1^2 + 2x_2^3$ by Newton's method.

   (a) Find the Hessian of $f(x)$.

   (b) Let $x^0 = (6, 6)$ be the initial solution. Run on iteration of Newton's method to find the next solution x^1.

   (c) Starting from x^1, run one more iteration of Newton's method to find the next solution x^2.

3. (10 points; 2 points each) Let's solve

$$\min_{x \in \mathbb{R}} \; f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 4x + 1.$$

   Let $x^0 = 2$ be the initial solution.

   (a) Find the gradient and Hessian of $f(x)$.

   (b) Run one iteration of gradient descent to get the next solution x^G. In each iteration, let the step size be that bringing you to the global minimum along the improving direction.

   (c) Run one iteration of Newton's method to get the next solution x^F.