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Suggested Solution for Pre-lecture Problems for Lecture 13

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1. (a) $\nabla f(x) = (2x_1, 4x_2)$.

   (b) $\nabla f(x^0) = (2, 0)$.
   $a_0 = \text{argmin}_{a \geq 0} f(x^0 - a\nabla f(x^0)) = \text{argmin}_{a \geq 0} f(1 - 2a, 0) = \text{argmin}_{a \geq 0}(1 - 2a)^2 \Rightarrow a_0 = \frac{1}{2}$.
   $x^1 = x^0 - a_0 \nabla f(x^0) = (1, 0) - \frac{1}{2}(2, 0) = (0, 0)$.

   (c) Since $||\nabla f(x^1)|| = ||(0, 0)|| = 0$, we stop the iteration and conclude that x^1 is our solution.

2. (a) $\nabla f(x) = \begin{bmatrix} 2x_1 \\ 6x_2^2 \end{bmatrix}$ and $\nabla^2 f(x) = \begin{bmatrix} 2 & 0 \\ 0 & 12x_2 \end{bmatrix}$.

   (b) $\nabla f(x^0) = \begin{bmatrix} 12 \\ 216 \end{bmatrix}$ and $\nabla^2 f(x^0) = \begin{bmatrix} 2 & 0 \\ 0 & 72 \end{bmatrix}$.
   $x^1 = x^0 - \left[\nabla^2 f(x^0)\right]^{-1} \nabla f(x^0) = \begin{bmatrix} 6 \\ 6 \end{bmatrix} - \frac{1}{144} \begin{bmatrix} 72 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 12 \\ 216 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

   (c) $\nabla f(x^1) = \begin{bmatrix} 0 \\ 18 \end{bmatrix}$ and $\nabla^2 f(x^0) = \begin{bmatrix} 2 & 0 \\ 0 & 36 \end{bmatrix}$.
   $x^2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix} - \frac{1}{72} \begin{bmatrix} 36 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 18 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{5}{2} \end{bmatrix}$.

3. (a) $f'(x) = x^3 - x^2 - 2x + 4$ and $f''(x) = 3x^2 - 2x - 2$.

   (b) $f'(x^0) = 4$.
   $a_0 = \text{argmin}_{a \geq 0} f(2 - 4a)$, numerically we may find that the global minimum of a is $a_0 = 0.915$.
   $x^G = 2 - 0.915 \times 4 = -1.66$.

   (c) $f''(x^0) = 6$.
   $x^F = 2 - \frac{4}{6} = \frac{4}{3}$.