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Suggested Solution for Pre-lecture Problems for Lecture 4

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1. Let x_{ij} = oz of chemical i used to produce drug j, $i = 1, 2, j = 1, 2$.

$$\begin{aligned}
\max \quad & 6\left(x_{11}+x_{21}\right)+5\left(x_{12}+x_{22}\right)-6\left(x_{11}+x_{12}\right)-4\left(x_{21}+x_{22}\right) \\
\text{s.t.} \quad & x_{11} \geq 0.6\left(x_{11}+x_{21}\right) \\
& x_{22} \geq 0.5\left(x_{12}+x_{22}\right) \\
& x_{11}+x_{21} \leq 100 \\
& x_{12}+x_{22} \leq 90 \\
& x_{11}+x_{12} \leq 130 \\
& x_{21}+x_{22} \leq 80 \\
& x_{ij} \geq 0 \quad \forall i=1,2, j=1,2.
\end{aligned}$$

2. Let $w = \min\{x_1, x_2\}$. The linearized LP is

$$\begin{aligned}
\max \quad & 5w+3x_2 \\
\text{s.t.} \quad & w \leq x_1 \\
& w \leq x_2 \\
& x_1 \leq 16 \\
& x_2 \leq 16 \\
& x_1+4x_2 \leq 20 \\
& x_2 \geq 8 \\
& x_i \geq 0 \quad \forall i=1,2.
\end{aligned}$$

3. Let $s = \min\{x_1, x_2 + x_3\}$, $t = \max\{x_2, x_1 + x_3\}$, $u = \min\{x_1, x_2 + 4\}$, $v = \max\{x_1, 4x_2 - x_3, 6\}$. The linearized LP is

$$\begin{aligned}
\max \quad & 5s-3t \\
\text{s.t.} \quad & s \leq x_1 \\
& s \leq x_2+x_3 \\
& t \geq x_2 \\
& t \geq x_1+x_3 \\
& x_1 \geq 16-x_1 \\
& x_1 \geq x_1-16 \\
& u \leq x_1 \\
& u \leq x_2+4 \\
& v \geq x_1 \\
& v \geq 4x_2-x_3 \\
& v \geq 6 \\
& u \geq v \\
& x_i \geq 0 \quad \forall i=1,2,3.
\end{aligned}$$