Operations Research, Spring 2017

Pre-lecture Problems for Lecture 9: Single-variate Nonlinear Programming

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Note. The deadline of submitting the pre-lecture problem is ***9:20 am, April 27, 2015***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Determine whether the following sets and functions are convex:
   (a) $\{(x_1, x_2) \in \mathbb{R}^2 | x_1 + x_2 \geq 4, x_1 \geq 0, x_2 \leq 0\}$.
   (b) $\{(x_1, x_2) \in \mathbb{R}^2 | x_1^2 + x_2^2 \geq 4, x_1 \geq 0, x_2 \leq 0\}$.
   (c) $f(x) = 2x^3 - x^2 - 2x + 1$ for $x \in \mathbb{R}$.
   (d) $f(x) = \begin{cases} -x & \text{if } x < 1 \\ -1 & \text{if } x \geq 1 \end{cases}$.

2. (0 point) Analytically find a global minimum for the following functions:
   (a) $f(x) = 3x^2 + 2x + 1$ for $x \in \mathbb{R}$.
   (b) $f(x) = 2x^3 - x^2 - 2x + 1$ for $x \in [-1, \infty)$.

3. (10 point) A retailer prices a single product. If the price is set to p, the demand of this price will be
$$D(p) = \begin{cases} 160 - 2p & \text{if } p \in [0, 40] \\ 120 - p & \text{if } p \in [40, 120] \\ 0 & \text{if } p \in (120, \infty) \end{cases}.$$
   The unit purchasing cost for the product is 10. The retailer tries to find a price that maximizes its profit.
   (a) (4 points) Formulate the retailer's problem as a nonlinear program.
   (b) (2 points) If the retailer is restricted to choose its price below 40, what is an optimal price?
   (c) (2 points) If the retailer is restricted to choose its price above 40, what is an optimal price?
   (d) (2 points) Solve the retailer's problem by combining your findings in Parts (b) and (c).