Operations Research, Spring 2017

Pre-lecture Problems for Lecture 12: Multi-variate Nonlinear Programming

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Note. The deadline of submitting the pre-lecture problem is ***9:20 am, May 4, 2017***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) For each of the following matrices, determine whether it is positive semi-definite.
   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
   (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.
   (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
2. (0 point) For each of the following functions, find the region over which the function is convex.
   (a) $f(x) = x^3 + 2x^2 + x + 2$.
   (b) $f(x_1, x_2) = x_1^3 + 2x_2^2 + x_1 + 2$.
   (c) $f(x_1, x_2, x_3) = x_1^2 x_3 + 2x_2 x_3 + x_1 + 2$.
3. (10 points; 2 points each) Consider the following nonlinear program
$$\begin{aligned} \min \quad & (x_1 - 3)^2 + (x_2 - 2)^2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 4. \end{aligned}$$
   (a) Prove or disprove that the NLP is a convex program.
   (b) Find the Lagrangian of this NLP. What is the sign of your Lagrangian multiplier?
   (c) Formulate the Lagrangian relaxation.
   (d) According to the FOC of the Lagrangian, find a necessary condition for any optimal solution.
   (e) Find an optimal solution for the NLP.