

Operations Research, Spring 2017

Suggested Solution for Pre-lecture Problems for Lecture 12

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1. (a) $\nabla f(x) = (2x_1, 4x_2)$.
 (b) $\nabla f(x^0) = (2, 0)$.
 Let $a_0 = \operatorname{argmin}_{a_0 \geq 0} f(x^0 - a_0 \nabla f(x^0)) = \operatorname{argmin}_{a_0 \geq 0} f(1 - 2a_0, 0) = \operatorname{argmin}_{a_0 \geq 0} (1 - 2a_0)^2 \Rightarrow a_0 = \frac{1}{2}$.
 $x^1 = x^0 - a_0 \nabla f(x^0) = (1, 0) - \frac{1}{2}(2, 0) = (0, 0)$.
 (c) Since $\|\nabla f(x^1)\| = \|(0, 0)\| = 0$, we stop the iteration and conclude that x^1 is our solution.

2. (a) $\nabla f(x) = \begin{bmatrix} 2x_1 \\ 6x_2^2 \end{bmatrix}$ and $\nabla^2 f(x) = \begin{bmatrix} 2 & 0 \\ 0 & 12x_2 \end{bmatrix}$.
 (b) $\nabla f(x^0) = \begin{bmatrix} 12 \\ 216 \end{bmatrix}$ and $\nabla^2 f(x^0) = \begin{bmatrix} 2 & 0 \\ 0 & 72 \end{bmatrix}$.
 $x^1 = x^0 - [\nabla^2 f(x^0)]^{-1} \nabla f(x^0) = \begin{bmatrix} 6 \\ 6 \end{bmatrix} - \frac{1}{144} \begin{bmatrix} 72 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 12 \\ 216 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.
 (c) $\nabla f(x^1) = \begin{bmatrix} 0 \\ 18 \end{bmatrix}$ and $\nabla^2 f(x^0) = \begin{bmatrix} 2 & 0 \\ 0 & 36 \end{bmatrix}$.
 $x^2 = x^1 - [\nabla^2 f(x^1)]^{-1} \nabla f(x^1) = \begin{bmatrix} 0 \\ 3 \end{bmatrix} - \frac{1}{72} \begin{bmatrix} 36 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 18 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{5}{2} \end{bmatrix}$.

3. (a) $\nabla f(x) = x^3 - x^2 - 2x + 4$ and $\nabla^2 f(x) = 3x^2 - 2x - 2$.
 (b) $\nabla f(x^0) = 2$.
 Let $a_0 = \operatorname{argmin}_{a_0 \geq 0} f(x^0 - a_0 \nabla f(x^0)) = \operatorname{argmin}_{a_0 \geq 0} f(1 - 2a_0) = \operatorname{argmin}_{a_0 \geq 0} (4a_0^4 - \frac{16}{3}a_0^2 - 2a_0^2 - 4a_0 + \frac{47}{12}) \Rightarrow a_0 \approx 1.329$.
 $x^G = x^0 - a_0 \nabla f(x^0) = 1 - 1.329 \times 2 = -1.658$.
 (c) $\nabla^2 f(x^0) = -1$.
 $x^F = x^0 - [\nabla^2 f(x^0)]^{-1} \nabla f(x^0) = 1 - \frac{2}{-1} = 3$.