Common Mathematical Notations and Operations

Ling-Chieh Kung*

October 12, 2014

Note. Throughout this handout, we use x, y, and z to denote real numbers or vectors, n and m to denote integers, and i, j, and k to denote indices. R codes are written in font style `like this`. If you would like to add anything into the list, please let me know. Thank you.

1 Mathematical notations

- $\mathbb{N}$ is the set of all natural numbers (positive integers); $\mathbb{Z}$ is the set of all integers; $\mathbb{Q}$ is the set of all rational numbers (which can be written as the ratio of two integers); $\mathbb{R}$ is the set of all real numbers.
- ( ) is a pair of parentheses, [ ] is a pair of square brackets, and { } is a pair of curly brackets.
- $[x, y]$ is the (closed) interval containing all real numbers between x and y, including x and y. We write $z \in [x, y]$ if $x \leq z \leq y$.
- (x, y) is the open interval containing all real numbers between x and y, excluding x and y. We write $z \in (x, y)$ if $x < z < y$.
- $[x, y)$ is the right open interval containing all real numbers between x and y, including x but excluding y. We write $z \in [x, y)$ if $x \leq z < y$.
- $(x, y]$ is the left open interval containing all real numbers between x and y, including y but excluding x. We write $z \in (x, y]$ if $x < z \leq y$.
- $\equiv$ is used for defining a notation. E.g., $\mu \equiv \frac{\sum_{i=1}^{N} x_i}{N}$ is the definition of population mean.
- A scalar is a single number; a vector is a sequence of numbers. Sometimes we write $x = (x_1, x_2, ..., x_n)$ to represent a vector of length n, where x_i is the ith element/number in vector x.

*Department of Information Management, National Taiwan University; lckung@ntu.edu.tw.

2 Mathematics operations

- $x + y$, read as "x plus y," means adding x and y to find their summation. E.g., $5 + 7$ is 12. In R, do this by typing `5 + 7`.

- $x - y$, read as "x minus y," means subtracting y from x to find their difference. E.g., $5 - 7$ is -2. In R, do this by typing `5 - 7`.

- xy or $x \times y$, read as "xy" or "x times y," means multiplying x by y to find their product. E.g., $5 \times 7 = 35$. In R, do this by typing `5 * 7`.

- $\frac{x}{y}$, read as "x divided by y" or "x over y," means dividing x by y to find the ratio of x to y. E.g., $\frac{6}{2} = 3$. In R, do this by typing `6 / 2`.

- $\text{mod}(n, m)$ is the operation for dividing n by m to find the remainder of this division. This is read as "x modulus y" by some people. E.g., $\text{mod}(10, 3) = 1$. In R, do this by typing `10 %% 3`.

- x^2, read as "x square," means multiplying x twice to find its square. E.g., $3^2 = 9$. In R, do this by typing `3 ^ 2`.

- x^3, read as "x cube," means multiplying x for three times to find its cube. E.g., $3^3 = 27$. In R, do this by typing `3 ^ 3`.

- x^n, read as "x to the power of n," means multiplying x for n times to find its nth power. E.g., 3^n is 243 if $n = 5$. In R, do this by typing `3 ^ n`.

- $\sqrt{x}$, read as "square root of x," means finding a number y such that $y^2 = x$. E.g., $\sqrt{9} = 3$. In R, do this by typing `sqrt(9)`.

- x_i, read just as "x i," means finding the ith element of vector x. E.g., if $x = (10, 11, 12)$, $x_2 = 11$. In R, do this by typing `x <- 10:12` and then `x[2]`.

- $\sum_{i=1}^{n} x_i$, read as "sum from x_1 to x_n," means to calculate $x_1 + x_2 + \cdots + x_n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=1}^{n} x_i = 33$. In R, do this by typing `x <- 10:12` and then `sum(x)`.[1]

- More generally, $\sum_{i=j}^{k} x_i$, read as "sum from x_j to x_k," means to calculate $x_j + x_{j+1} + \cdots + x_k$ for some numbers $j \geq 1$ and $k \leq n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=2}^{3} x_i = 23$. In R, do this by typing `x <- 10:12` and then `sum(x[2:3])`.[2]

- $\lfloor x \rfloor$, read as "floor of x,", means rounding down x to the closest integer no greater than x. E.g., $\lfloor 1.9 \rfloor = 1$. In R, do this by typing `floor(1.9)`.

- $\lceil x \rceil$, read as "ceiling of x," means rounding up x to the closest integer no less than x. E.g., $\lceil 1.1 \rceil = 2$. In R, do this by typing `ceiling(1.1)`.

[1] Here we have assumed that x has n elements.

[2] When we have enough spaces, we write $\displaystyle\sum_{i=j}^{k} x_i$.

- $|x|$, read as "the absolute value of x," means finding the distance between x and 0. E.g., $|-5| = 5$. In R, do this by typing `abs(-5)`.

- $n!$, read as "the factorial of n," means finding the product of all positive integers no greater than n. E.g., $3! = 3 \times 2 \times 1 = 6$. In R, do this by typing `factorial(3)`.

- $\max\{x, y\}$ or $\max(x, y)$, read as "the maximum of x and y," means finding the larger one between x and y. E.g., $\max\{1, 4\} = 4$. In R, do this by typing `max(x, y)`. When x is a vector, $\max\limits_{i=1,\ldots,n} \{x_i\}$ is the largest element in x. In R, do this by typing `max(x)`.

- $\min\{x, y\}$ or $\min(x, y)$, read as "the minimum of x and y," means finding the smaller one between x and y. E.g., $\min\{1, 4\} = 1$. In R, do this by typing `min(x, y)`. When x is a vector, $\min\limits_{i=1,\ldots,n} \{x_i\}$ is the smallest element in x. In R, do this by typing `min(x)`.

3 Common notations in statistics

- N is the population size and n is the sample size.

- $\mu \equiv \frac{\sum_{i=1}^{N} x_i}{N}$ (read as "miu") is the population mean and $\bar{x} \equiv \frac{\sum_{i=1}^{n} x_i}{n}$ (read as "x-bar") is the sample mean.

- $\sigma^2 \equiv \frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$ (read as "sigma square") is the population variance and $s^2 \equiv \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ (read as "s square") is the sample variance.

- $\sigma \equiv \sqrt{\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}}$ is the population standard deviation and $s \equiv \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$ is the sample standard deviation.