Statistics and Data Analysis

The Dice Game

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Three investments

- Let Green, Red, and White be three hypothetical **investments** with the following probability distributions for their yearly **gross returns**.

Probability	1/6	1/6	1/6	1/6	1/6	1/6
Green	0.8	0.9	1.1	1.1	1.2	1.4
Red	0.06	0.2	1	3	3	3
White	0.95	1	1	1	1	1.1

Returns and risks

- For each investment, we may find its **mean** (expected value) and **standard deviation**.

Probability	1/6	1/6	1/6	1/6	1/6	1/6	Mean	SD
Green	0.8	0.9	1.1	1.1	1.2	1.4	1.083	0.195
Red	0.06	0.2	1	3	3	3	1.710	1.323
White	0.95	1	1	1	1	1.1	1.008	0.045

The mean measures the expected **return**. The standard deviation measures the **risk**.

- Which one do you prefer?

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Investments and outcomes

- You have \$1000 invested in each of Green, Red, and White.
- The **market uncertainty** is determined by the outcome of rolling three dices.
  - The outcome determines the annual returns of the investments.
- Suppose the outcome is 2 for Green, 5 for Red, and 3 for White.

Die Value	1	2	3	4	5	6
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  Then the after-one-year values are \$900, \$3000, and \$1000, respectively.
- Suppose the outcome is 4 for Green, 2 for Red, and 6 for White. Then the after-two-year values are \$990, \$600, and \$1100, respectively.

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On the worksheet

- On the worksheet, investment amounts and dice values can be recorded.

Year		Green	Red	White
1	Start	1000 (2)	1000 (5)	1000 (3)
	End	900	3000	1000
2	Start	900 (4)	3000 (2)	1000 (6)
	End	990	600	1100
3	Start			
	End			

Adjusting investments

- In practice, one may **adjust the investments** before a year starts.
- Suppose that we have adjusted the amounts at the end of year 1: \$1600 for Green, \$1600 for Red, and \$1700 for White.

Year		Green	Red	White
1	Start	1000 (2)	1000 (5)	1000 (3)
	End	900	3000	1000
2	Start	1600 (4)	1600 (2)	1700 (6)
	End	1760	320	1870
3	Start			
	End			

- Objective: To **maximize the total value** at the end of year 8.

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Game procedure

- Form teams of 6 students.
- Start with $1000 in each investment. Carry out the game for 8 years.
- The instructor will role the dices for all teams.
- Team members discuss together for amount redistribution.
- Roles for team members:
  - Market: Write down the outcome of dice rolling and find the right gross return rates.
  - Accountant: Calculate the values at the end of a year.
  - Green investor: Double check the Green account.
  - Red investor: Double check the Red account.
  - White investor: Double check the White account.
  - CEO: Lead the team.

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Let's start!

Year 1

Year 2

Year 3

Year 4

Year 5

Year 6

Year 7

Year 8 (the final year)

End!

Discussions

- What is the best strategy in this game?

Discussions

- "Do not put all your eggs in one basket."

The "Pink" investment

- Consider the "Pink" investment (which is a **portfolio**):

$$\text{Pink} = \frac{\text{Red} + \text{White}}{2}.$$

- The mean and standard deviation of Pink can be calculated:

Investment	Mean	SD
Green	1.083	0.195
Red	1.710	1.323
White	1.008	0.045
Pink	1.359	0.662

Note that $\mu_{\text{pink}} = \frac{1}{2}\mu_{\text{red}} + \frac{1}{2}\mu_{\text{white}}$ but $\sigma_{\text{pink}} < \frac{1}{2}\sigma_{\text{red}} + \frac{1}{2}\sigma_{\text{white}}$!

- As we will introduce later in this semester, $\sigma_{\text{pink}} = \sqrt{\frac{1}{4}\sigma^2_{\text{red}} + \frac{1}{4}\sigma^2_{\text{white}}}$.

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Volatility-adjusted returns

- To compare two investments, we may compare their **volatility-adjusted returns**:

$$\text{Volatility-adjusted return} = \mu - \frac{\sigma^2}{2}.$$

Investment	Mean	SD	Variance	Volatility-adjusted return
Green	1.083	0.195	0.038	1.064
Red	1.710	1.323	1.750	0.835
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Lessons and warnings

- Lessons:
  - Expected values (means) and standard deviations (or variances) are used to measure returns and risks.
  - Diversification is a good idea to maximize long-term returns.
  - To look for the best diversification, probability helps.
- Warnings:
  - Knowing the probability **distributions** is hard.
  - Performances of multiple investments may actually be **dependent**.
- Responses:
  - Estimating the distributions is easier than predicting the outcome.
  - There are methods to address dependency (through covariances).

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