Statistics and Data Analysis, Fall 2016
Pre-lecture Problems 5

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note 1. The deadline of submitting the pre-lecture problem is **_14:30, October 19, 2016_**. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

Note 2. Please make your answer as clear (i.e., easy to read) as possible. We reserve the right to take away points when the correctness cannot be easily determined (e.g., when the writing is messy and cannot be easily understood).

1. (0 points) Let X be the number of heads obtained after tossing three coins, where the probability of getting a head from each coin is p. Find the probability distribution of X.

2. (0 points) Let $X \sim \text{ND}(100, 20)$ and $Y \sim \text{ND}(80, 10)$. Let $F_X(\cdot)$ and $F_Y(\cdot)$ be the cdf of X and Y, respectively. Answer the following true/false questions:

   (a) $\mathbb{E}[X] > \mathbb{E}[Y]$.
   (b) $F_X(80) < \frac{1}{2}$.
   (c) $1 - F_Y(70) > \frac{1}{2}$.
   (d) $F_X(80) + F_Y(90) = 1$.

3. (10 points; 2 points each) Consider a continuous random variable X with the following pdf

$$f(x) = 2x$$

   over the sample space $S = [0, 1]$.

   (a) Depict the pdf.
   (b) Find $\Pr(X \leq 0.5)$.[1]
   (c) Find $\Pr(0.3 \leq X \leq 0.7$.
   (d) Find the cdf $F(x) = \Pr(X \leq x)$.
   (e) Robin makes the following statement: "As $f(1) = 2$, the probability for X to be 1 is 2." Note that this is certainly wrong: No event can has an occurring probability that is above 1. Please indicate where Robin makes a mistake.

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[1] A right triangle's area is one half of the product of the lengths of the two legs. For more details, see, e.g., `https://en.wikipedia.org/wiki/Right_triangle`.