Statistics and Data Analysis
Probability

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1. A lottery ticket costs \$10. Possible outcomes and their probabilities are: With probability 0.01, you win \$1000; with probability 0.05, you win \$100; with probability 0.1, you win \$10.

   (a) Let X be the amount of money that you will win. What is the sample space of X?

   (b) Construct a table to represent the distribution of X.

   (c) You have decided that you will buy the ticket if your expected earning is larger than the ticket price. Should you buy the ticket?

2. Let X be the number of people who visit a particular web page in the next hour.

   (a) Suppose that the distribution of X is estimated to be

x	50	150	250	350	450	550	650
$\Pr(X = x)$	0.3	0.2	0.2	0.1	0.1	0.05	0.05

   Find $\mu = \mathbb{E}[X]$, the expected number of next-hour visitors.

   Note. The MS Excel sheet "Given X's distribution" contains the distribution information.

   (b) Find $\sigma^2 = \mathrm{Var}(X)$, the variance of the next-hour visitors.

3. On a web page, there is a slot for display advertisement. Let X be the number of next-hour visitor to this page, whose distribution is

x	50	150	250	350	450	550	650
$\Pr(X = x)$	0.3	0.2	0.2	0.1	0.1	0.05	0.05

Suppose that the *click-through rate* (CTR) is 0.02, i.e., given any customer, the probability for her to click the advertisement is 2%. That CTR is identical for everyone.

(a) Let Y be the number of customers who will click the advertisement. How would you find the distribution of Y? Is it easy?

(b) Find $\mathbb{E}[Y]$, the expected value of Y. How do you find it from $\mathbb{E}[X]$?

4. Consider a random variable X whose pdf is

$$f(x) = \begin{cases} \frac{4}{3}x & \text{if } 0 \le x \le 1 \\ 4 - \frac{8}{3}x & \text{if } 1 < x \le \frac{3}{2} \end{cases}.$$

(a) Draw the pdf. Does $f(1) = \frac{4}{3}$ mean $\Pr(X = 1) = \frac{4}{3}$?

(b) Find $\Pr(X \le \frac{1}{2})$.

(c) Find $\Pr(X \ge 1)$.

(d) Show $\Pr(X \le \frac{3}{2}) = 1$. Is this a coincidence?

5. Let D be the daily demand of a certain product. It is typical to use a normal distribution to approximate the distribution of D. Let $D \sim \text{ND}(100, 20)$, i.e., D is normally distributed with mean 100 and standard deviation 20.

(a) Find $\Pr(D \leq 100)$ without using software.

(b) Find $\Pr(D \leq 90)$.
(In MS Excel: `NORM.DIST()`)

(c) Find $\Pr(D \leq 82)$.

(d) Find $\Pr(D \geq 96)$.

(e) Find $\Pr(110 \leq D \leq 130)$.

(f) Find $\Pr(D \leq 70) + \Pr(D \geq 130)$. Compare it with $2\Pr(D \leq 70)$.

6. Let $D \sim \mathrm{ND}(100, 20)$ be the daily demand of a certain product.

(a) Find a value q_1 such that $\Pr(D \leq q_1) = 0.4$.
(In MS Excel: `NORM.INV()`)

(b) Find a value q_2 such that $\Pr(D \leq q_2) = 0.6$. Is $q_2 = 200 - q_1$? Why or why not?

(c) Find an order quantity q that achieves 90% of service level for the next day, i.e., the probability to have no shortage in a day is 90%.

(d) For service levels 10%, 20%, ..., and 90%, find the corresponding order quantities. Plot them to illustrate how these quantities changes as the desired service level increases.

7. Let $X \sim \text{ND}(30, 5)$, $Y \sim \text{ND}(10, 2)$, and $Z \sim \text{ND}(0, 1)$. Note that Z is a standard normal random variable.

(a) Find $\Pr(X \leq 25)$, $\Pr(Y \leq 8)$, and $\Pr(Z \leq -1)$. Show that they are all the same.

(b) In MS Excel, use `NORM.S.DIST()` to calculat $\Pr(Z \leq -1)$. Then use `NORM.S.INV()` to find z such that $\Pr(Z \leq z) = 0.16$.