

Statistics I, Fall 2012

Homework 10

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

- (50 points) Write down your name.
- (10 points; 5 points each) Suppose that from an exponential population with rate $\lambda > 2$ we generate a random sample $\{X_1, X_2, X_3\}$. Obviously, the sample size is 3. Consider the following five estimators of the rate λ :

$$\hat{\lambda}_1 = X_1, \quad \hat{\lambda}_2 = \frac{X_1 + X_2}{2}, \quad \hat{\lambda}_3 = \frac{X_1 + 2X_2 + 3X_3}{12}, \quad \hat{\lambda}_4 = e^{X_3}, \quad \text{and} \quad \hat{\lambda}_5 = \bar{X}.$$

- Which of these estimators are unbiased?
- Among these unbiased estimators, let $\hat{\lambda}_i$ be the one with the lowest variance and $\hat{\lambda}_j$ be the one with the highest variance? What are the values of i and j ? What is the efficiency of $\hat{\lambda}_i$ relative to $\hat{\lambda}_j$?

Note. You may verify that $\hat{\lambda}_3$ has a smaller variance than $\hat{\lambda}_5$. Should we prefer $\hat{\lambda}_3$ over $\hat{\lambda}_5$?

- (10 points; 2.5 points each) Answer the following true/false questions regarding estimating the population mean when the population variance is known. For each question, assume that all conditions except the one mentioned in the question remain fixed. DO NOT provide your reasons.
 - In a finite population case, when the population size becomes smaller, the confidence interval will become smaller.
 - When a larger confidence level is required, to maintain the same confidence interval, a larger sample size is required.
 - When one increase the sample size, a confidence interval will become smaller.
 - When the population is binomial, because a binomial distribution can be approximated by a normal distribution, we do not need the sample size to be larger than 30 to use the z distribution for constructing a confidence interval.
- (10 points, 5 points each; modified from Problem 8.6 in the textbook) A candy company fills a 20-ounce package of Halloween candy with individually wrapped pieces of candy. The number of pieces of candy per package varies because the package is sold by weight. The company wants to estimate the number of pieces per package. The population standard deviation is known to be 0.87.
 - Suppose inspectors randomly sample 120 packages and count the number of pieces in each package. The data are recorded in Column A in the sheet "Candy" of the MS Excel file "StatFa12_hw10.xlsx". Construct a 95% confidence interval to estimate the average number of pieces per package for the population.
 - Suppose inspectors randomly sample 240 packages and count the number of pieces in each package. The data are recorded in Column B in the sheet "Candy" of the MS Excel file "StatFa12_hw10.xlsx". Construct a 95% confidence interval to estimate the average number of pieces per package for the population.
- (20 points, 5 points each; modified from Problem 8.10 in the textbook) A meat-processing company in the Midwest in the United States produces a package of eight small sausage sandwiches. The product is sold to 100 retailers around the country. While each retailer determines the retail price it will charge for this item, the company is interested in knowing the average retail price charged for this item in retail stores across the country. A researcher for the company contacts 18 of these retailers and ascertains the selling prices for the product. The price data are recorded in the sheet "Meet" of the MS Excel file "StatFa12_hw10.xlsx". The population variance is 0.0121.

- (a) What assumption regarding the population is needed to construct a confidence interval for estimating the average retail price? Answer the following questions by assuming that the assumption you need is true.
- (b) Construct a 90% confidence interval to estimate this price.
- (c) Construct a 99% confidence interval to estimate this price.
- (d) Suppose the owner of the meat-processing company is wondering whether the average retail price is below \$2.18. If so, it will be better for him to raise the wholesale price. Based on your answers in Part (b), make a suggestion to the owner on whether the wholesale price should be increased.