Statistics I, Fall 2012
Homework 13

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Note. For each problem, define the notations you use, if any.

1. (25 points; 5 points each) For the population mean μ, consider the following hypothesis

$$H_0 : \mu = 7.5$$
$$H_a : \mu < 7.5.$$

with the sample mean $\bar{x} = 6.91$, sample size $n = 36$, population standard deviation $\sigma = 1.21$, and significance level $\alpha = 0.01$. Assume the population is normal.

   (a) What is the probability of making a Type I error?

   (b) If $\mu = 7.2$, what is the probability of making a Type II error?

   (c) If $\mu = 7$, what is the probability of making a Type II error?

   (d) If $\mu = 7$ but $\alpha = 0.05$, what is the probability of making a Type II error?

   (e) Depict the power of this test as a function of μ over the domain $[5.5, 7.5]$.

2. (15 points; modified from Problem 9.5) According to the U.S. Bureau of Labor Statistics, the average weekly earnings of a production worker in 1997 were \$424.20. Suppose a labor researcher wants to test to determine whether this information is still accurate today. The researcher randomly selects 70 production workers from across the United States and obtains a representative earnings statement for one week from each. The resulting sample average is \$432.70. Assuming a population standard deviation of \$32 and a 5% level of significance, determine whether the mean weekly earnings of a production worker have changed.

3. (15 points; modified from Problem 9.8) According to a report released by CIBC entitled "Women Entrepreneurs: Leading the Charge," the average age for Canadian businesswomen in 2008 was 42. In the report, there was some indication that researchers believed that this mean age will increase. Suppose now, a few years later, business researchers in Canada want to test to determine if, indeed, the mean age of a Canadian businesswoman has increased. The researchers randomly sample 97 Canadian businesswomen and ascertain that the sample mean age is 44.4. From past experience, it is known that the population standard deviation is 8.25. Test to determine if the mean age of a Canadian businesswoman has increased, using a 1% level of significance. What is the p-value for this test? What is the decision?

4. (15 points; modified from Problem 9.9) According to HowtoAdice.com, the average price charged to a customer to have a 12' by 18' wall-to-wall carpet shampoo cleaned is about \$50. Suppose that a start-up carpet-cleaning company believes that in the region in which they operate, the average price for this service is higher. To test this hypothesis, the carpet-cleaning company randomly contacts 25 customers who have recently had a 12' by 18' wall-to-wall carpet shampoo cleaned and asked the customers how much they were charged for the job. Suppose the resulting data are given in the sheet "9.9" of the MS Excel file "StatFa12_hw13.xlsx" and that the population standard deviation price is \$3.25. Use a 10% level of significance to test their hypothesis. Assume that such prices are normally distributed in the population. What is the observed value? What is the p-value? What is the decision?

5. (15 points; modified from Problem 9.15) A hole-punch machine is set to punch a hole 1.9 centimeters in diameter in a strip of sheet metal in a manufacturing process. The strip of metal is then creased and sent on to the next phase of production, where a metal rod is slipped through the hole. It is important that the hole be punched to the specified diameter of 1.9 cm. To test punching accuracy,

technicians have randomly sampled 10 punched holes and measured the diameters. The data (in centimeters) are given in the sheet "9.15" of the MS Excel file "StatFa12_hw13.xlsx". Use an alpha of 10% to determine whether the holes are being punched an average of 1.9 centimeters. Assume the punched holes are normally distributed in the population.

6. (15 points; modified from Problem 9.18) Major cities around the world compete with each other in an effort to attract new businesses. Some of the criteria that businesses use to judge cities as potential locations for their headquarters might include the labor pool; the environment, including work, governmental, and living; the tax structure, the availability of skilled/educated labor, housing, education, medical care; and others. Suppose in a study done several years ago, the city of Atlanta received a mean rating of 3.51 (on a scale of 1 to 5) on housing, but that since that time, considerable residential building has occurred in the Atlanta area such that city leaders feel the mean might now be higher. They hire a team of researchers to conduct a survey of businesses around the world to determine how businesses now rate the city on housing (and other variables). Sixty-four businesses take part in the new survey, with a result that Atlanta receives a mean response of 3.71 on housing with a sample standard deviation of 0.65. Assuming that such responses are normally distributed, use a 1% level of significance and these data to test to determine if the mean housing rating for the city of Atlanta by businesses has significantly increased.

7. (15 points; modified from Problem 9.19) Based on population figures and other general information on the U.S. population, suppose it has been estimated that, on average, a family of four in the United States spends about $1,200 annually on dental expenditures. Suppose further that a regional dental association wants to test to determine if this figure is accurate for their area of the country. To test this, 25 families of four are randomly selected form the population in that area of the country and a log is kept of the family's dental expenditures for one year. The resulting data are given in the sheet "9.19" of the MS Excel file "StatFa12_hw13.xlsx". Assuming that dental expenditures are normally distributed in the population, use the data and an alpha of .05 to test the dental association's hypothesis.