Information Economics

Fall 2013

Suggested Solution for Homework 3

1. When $\theta = \frac{1}{2}$, the payoff matrix is

	I	D
I	$4/9$, $4/9$	$256/441$, $5/21$
D	$5/21$, $256/441$	$35/108$, $35/108$

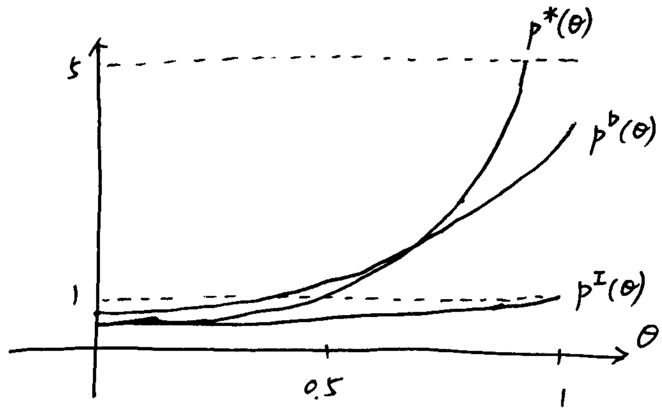
, or

	I	D
I	0.44, 0.44	0.58, 0.24
D	0.24, 0.58	0.32, 0.32

. It is clear that (I, I) is the unique equilibrium.

2. (a) We solve $\max_p \; 2p(1 - p + \theta p)$, whose optimal solution is $p^*(\theta) = \frac{1}{2(1-\theta)}$

(b) The three curves are



$p^*(\theta) > p^D(\theta)$ when θ is small and $p^D(\theta) > p^*(\theta)$ when θ is large.

(c) The only value of θ under which $p^D(\theta) = p^*(\theta)$ can be found by solving
$$\frac{2(3-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)} = \frac{1}{2(1-\theta)} \iff 4 - 6\theta - \theta^2 + 2\theta^3 = 0.$$

Numerically we may find a unique within-zero-and-one root 0.6991.

Analytically we can show that the polynomial has exactly one root within 0 and 1.

<u>Note</u> When θ is small, decentralization not only drives the prices up. It makes the prices too high!

3 (a) The worker solves $\max_{a \geq 0} \; t - \frac{1}{2}a^2$ and get the optimal service level $a^* = 0$.

Having this in mind, the retailer solves $\max_{p,t} \; p(1-p) - t$, where the $\text{s.t.} \; t \geq 0$ constraint induces participation. The optimal solution is $t^* = 0$ and $p^* = \frac{1}{2}$.

The retailer earns $\frac{1}{4}$ and the worker earns 0.

(b) $\max_{a \geq 0} \; t + vp(1-p+a) - \frac{1}{2}a^2 \Rightarrow a^* = vp$. He earns $t + vp(1-p) + \frac{v^2p^2}{2}$

(c) The retailer solves

$$\max_{p,t,v} \; p(1-p+vp)(1-v) - t$$
$$\text{s.t.} \; t + vp(1-p) + \frac{v^2p^2}{2} \geq 0.$$

At optimality the constraint must be binding, so she solves

$$\max_{p,v} \; p(1-p+vp)(1-v) + vp(1-p) + \frac{v^2p^2}{2} = \max_{p,v} \; p - p^2 + vp^2 - \frac{v^2p^2}{2}$$

$$\Rightarrow \begin{cases} 1 - 2p^* + 2vp^* - (v^*)^2 p^* = 0 \\ (p^*)^2 - v^*(p^*)^2 = 0 \end{cases} \Rightarrow (p^*, v^*) = (1, 1).$$
$$\Rightarrow t^* = -\frac{1}{2}$$

The retailer earns $\frac{1}{2}$. The worker earns 0.

(d) It makes both players (at least weakly) better off. The retailer earns more and the worker earns the same.

(e) $\max_{p,a} \; p(1-p+a) - \frac{1}{2}a^2 \Rightarrow \begin{cases} 1 - 2p^* + a^* = 0 \\ p^* - a^* = 0 \end{cases} \Rightarrow p^* = a^* = 1$.

The system profit is $\frac{1}{2}$.

(f) The contract with commission is efficient: The equilibrium price, service level, and system profit are all efficient. The reason is the following: The retailer can set $v = 1$ to induce the efficient service level and she will be willing to do that because the transfer t allows her to extract surplus from the worker.