IM 7011: Information Economics

Incentives in Decentralized Systems (Part 1)
Lecture 3.1: Incentive Misalignment and Double Marginalization

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When centralization is impossible

- We hope people all cooperate to maximize social welfare and then fairly allocate payoffs.
- Complete **centralization**, or **integration**, is the best.
- However, it is impossible.
  - Each person has her/his **self interest**.
- Facing a **decentralized** system, we will not try to integrate it.
  - We will not assume (or try to make) that people act for the society.
  - We will assume that people are all **selfish**.
  - Then we seek for **mechanisms** improve the efficiency.
  - This is the field of **mechanism design**.

Issues under decentralization

- What issues arise in a decentralized system?
- The **incentive** issue:
  - Workers need incentives to work hard.
  - Students need incentives to keep labs clean.
  - Manufacturers need incentives to improve product quality.
  - Users need incentives to keep using a social network.
- The **information** issue:
  - Efforts of workers and students are hidden.
  - Product quality and willingness-to-use are hidden.
- Information issues **amplify** or even **create** incentive issues.

Incentive alignment

- The typical goal is to **align** the incentives of different players.
- As an example, an employer wants her workers to work as hard as possible, but a worker always prefers vacations to works.
- There may be **incentive misalignment** between the employer and the employee.
- To better align their incentives, the employer may put what she cares into the employee's utility function.
- This is why we see sales bonuses and commissions!

Double marginalization

- In a supply chain or distribution channel, incentive misalignment may cause **double marginalization**.
- Consider the pricing in a supply chain problem:
  - The unit cost is c.
  - The manufacturer charges $w^* > c$ with one layer of "marginalization".
  - The retailer charges $r^* > w^*$ with another layer of marginalization.
  - The equilibrium retail price r^* is **too high**. Both firms are hurt.
- Consider the indirect newsvendor problem:
  - The unit cost is c.
  - The manufacturer charges $w^* > c$ with one layer of "marginalization".
  - The retailer orders a quantity q^* to maximize its expected profit. This is another layer of marginalization.
  - The equilibrium inventory level q^* is **too low**. Both firms are hurt.
- The two systems are both **inefficient** because the equilibrium decisions (retail price or inventory level) are **system-suboptimal**.

What should we do?

- How to reduce inefficiency?
- Complete integration is the best but impractical.
- We may make these player **interact differently**.
  - We may change the "game rules".
  - We may design different mechanisms.
  - All we want is to **induce satisfactory behaviors**.
  - Recall the ultimatum game!
- In these two lectures, we will introduce two seminal papers that show some ways to enhance efficiency.
  - Pasternack (1985): To change the **contract format**.
  - McGuire and Staelin (1983): To change the **channel structure**.

  Both were published in *Marketing Science*.

References

McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(1) 115–130.

Pasternack, B. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science* **4**(2) 166–176.

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Incentives in Decentralized Systems (Part 1)
Lecture 3.2: Return Contracts: Motivation, Example, and Model

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Introduction

- Pasternack (1985) studies a common practice adopted in distribution channels: **return** (buy-back) contracts.[1]
  - Why people use return contracts?
  - What is the benefit of using return contracts?
- In this lecture, we illustrate the main insights of Pasternack (1985) by a **simplified** model. One is encouraged to read the paper afterwards.
  - Different notations may be adopted to facilitate understanding.
- We will mostly adopt this way in this semester.

[1] Pasternack, B. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science* **4**(2) 166–176.

Road map

- **Motivation**.
- Example.
- Model.

Why return contracts?

- In many distribution channels, the manufacturer signs a **wholesale contract** with the retailer.
  - What happened in the indirect newsvendor problem?
  - The inventory level (order/production/supply quantity) is **too low**.
  - The inventory level is optimal for the retailer but too low for the system.
- Why the retailer orders an inefficiently low quantity?
- Demand is uncertain:
  - The retailer takes all the **risks** while the manufacturer is **risk-free**.
  - When the unit cost increases (from c to w), overstocking becomes more harmful. The retailer thus lower the inventory level.
- How to induce the retailer to order more?
  - Reducing the wholesale price? No way!
  - A practical way is for the manufacturer to **share the risk**.

Why return contracts?

- A **return** (buy-back) contract is a **risk-sharing** mechanism.
- When the products are not all sold, the retailer is allowed to return (all or some) unsold products to get credits.
- Contractual terms:
  - w is the wholesale price.
  - r is the buy-back price (return credit).
  - R is the percentage of products that can be returned.
- Several alternatives:
  - Full return with full credit: $R = 1$ and $r = w$.
  - Full return with partial credit: $R = 1$ and $r < w$.
  - Partial return with full credit: $R < 1$ and $r = w$.
  - Partial return with partial credit: $R < 1$ and $r < w$.
- In practice, the manufacturer may pay the retailer without asking for the physical goods. Why?

Pros and cons of return contracts

- Bad news 1: A return contract is harder to design.
- Bad news 2: A bad return contract may be worse than a good wholesale contract.
- Good news 1: A wholesale contract is a return contract.
  - Given any wholesale contract, setting $r = 0$ creates an equivalent return contract.
- Good news 2: A good return contract can be **win-win**.
- Good news 3: A well-designed return contract can be **efficient**.
- Before we jump into the analytical model, let's get the idea with a numerical example.

Road map

- Motivation.
- **Example**.
- Model.

A numerical example

- Consider a distribution channel in which a manufacturer (she) sells a product to a retailer (he), who then sells to end consumers.
- Suppose that:
  - The unit production cost is \$10.
  - The unit retail price is \$50.
  - The random demand follows a uniform distribution between 0 and 100.

Benchmark: integration

- As a benchmark, let's first find the **efficient inventory level**, which will be implemented when the two firms are integrated.
- Let Q_T^* be the efficient inventory level that maximizes the expected system profit, we have

$$\frac{Q_T^*}{100} = 1 - \frac{10}{50} \quad \Rightarrow \quad Q_T^* = 80.$$

- The expected system profit, as a function of Q, is

$$\begin{aligned}\pi_T(Q) &= 50\left\{ \int_0^Q x\left(\frac{1}{100}\right)dx + \int_Q^{100} Q\left(\frac{1}{100}\right)dx \right\} - 10Q \\ &= -\frac{1}{4}Q^2 + 40Q.\end{aligned}$$

- The optimal system profit is $\pi_T^* = \pi_T(Q_T^*) = \$1600$.

Wholesale contract

- Under the wholesale contract, we have the indirect newsvendor problem.
- We know that in equilibrium, the manufacturer sets the wholesale price $w^* = \frac{50+10}{2} = 30$ and the retailers orders $Q_R^* = 40$.
- The retailer's expected profit, as a function of Q, is

$$\begin{aligned}\pi_R(Q) &= 50\left\{\int_0^Q x\left(\frac{1}{100}\right)dx + \int_Q^{100} Q\left(\frac{1}{100}\right)dx\right\} - 30Q \\ &= -\frac{1}{4}Q^2 + 20Q.\end{aligned}$$

- The retailer's expected profit is $\pi_R^* = \pi_R(Q_R^*) = \$400$.
- The manufacturer's expected profit is $\pi_M^* = 40 \times (30 - 10) = \$800$.
- The expected system profit is $\pi_R^* + \pi_M^* = \$1200 < \pi_T^* = \$1600$.

Return contract 1

- Consider the following return contract:
  - The wholesale price $w = 30$.
  - The return credit $r = 5$.
  - The percentage of allowed return $R = 1$.
- The retailer's expected profit, as a function of Q, is

$$\pi_R^{(1)}(Q) = 50\left\{ \int_0^Q \frac{x}{100}dx + \int_Q^{100} \frac{Q}{100}dx \right\} + 5\int_0^Q \frac{Q-x}{100}dx - 30Q$$
$$= -\frac{1}{4}Q^2 + \frac{1}{40}Q^2 + 20Q \quad \Rightarrow \quad Q_R^{(1)} = \frac{400}{9} \approx 44.44.$$

- The retailer's expected profit is $\pi_R^{(1)} = \pi_R(Q_R^{(1)}) \approx \$444.44 > \pi_R^*$.
- The manufacturer's expected profit is $\pi_M^{(1)} = (\frac{400}{9})(30-10) - \frac{4000}{81} \approx 888.89 - 49.38 = \$839.51 > \pi_M^*$.
- The expected system profit is $\pi_R^{(1)} + \pi_M^{(1)} = \$1283.95 < \pi_T^* = \$1600$.

Return contract 2

- Consider a more generous return contract:
  - The wholesale price $w = 30$.
  - The return credit $r = 10$.
  - The percentage of allowed return $R = 1$.
- The retailer's expected profit, as a function of Q, is

$$\pi_R^{(2)}(Q) = 50\left\{ \int_0^Q \frac{x}{100} dx + \int_Q^{100} \frac{Q}{100} dx \right\} + 5\int_0^Q \frac{Q-x}{100} dx - 30Q$$
$$= -\frac{1}{4}Q^2 + \frac{1}{20}Q^2 + 20Q \quad \Rightarrow \quad Q_R^{(2)} = 50.$$

- The retailer's expected profit is $\pi_R^{(2)} = \pi_R(Q_R^{(2)}) = \$500 > \pi_R^{(1)}$.
- The manufacturer's expected profit is $\pi_M^{(2)} = 50 \times (30 - 10) - 125 \approx 1000 - 125 = \$875 > \pi_M^{(1)}$.
- The expected system profit is $\pi_R^{(2)} + \pi_M^{(2)} = \$1375 < \pi_T^* = \$1600$.

Comparison

- The **performance** of these contracts:

(w, r, R)	Q	π_R	π_M	$\pi_R + \pi_M$
$(30, 0, 1)$	40	400	800	1200
$(30, 5, 1)$	44.44	444.44	839.51	1283.95
$(30, 10, 1)$	50	500	875	1375

  - Will Q keep increasing when r increases?
  - Will π_R and π_M keep increasing when r increases?
  - Will $Q = Q_T^* = 80$ for some r? Will $\pi_R + \pi_M = \pi_T^* = 1600$ for some r?
- There are so many questions!
  - What if $w \neq 30$? What if $R < 1$?
  - What if the demand is not uniform?
- The main question: When may we achieve **channel coordination**, i.e., the retailer is induced to order the system-optimal quantity 80?
- We need a general analytical model to really deliver insights.

Road map

- Motivation.
- Example.
- **Model**.

Model

- We consider a manufacturer-retailer relationship in an indirect channel.
- The product is perishable and the single-period demand is random.
- Production is under MTO and the retailer is a newsvendor.
- We use the following notations:

Symbol	Meaning
c	Unit production cost
w	Unit wholesale price
r	Unit return credit
R	Percentage of allowed return
Q	Order quantity
F	Distribution function of demand
f	Density function of demand

- Assumptions:
  - $c < w < p$.
  - $r \leq w$.
  - $f(x) = 0$ for all $x < 0$.

Utility functions

- Under the return contract, the retailer's expected profit is

$$\begin{aligned}\pi_R(Q) = &- Qw \\ &+ \int_0^{(1-R)Q} (xp + RQr) f(x) dx \\ &+ \int_{(1-R)Q}^{Q} \Big[xp + (Q - x) r \Big] f(x) dx \\ &+ \int_Q^{\infty} Qp f(x) dx.\end{aligned}$$

- The manufacturer's expected profit is

$$\pi_M(Q) = Q(w - c) - \int_0^{(1-R)Q} RQr f(x) dx - \int_{(1-R)Q}^{Q} (Q - x) r f(x) dx.$$

- The expected system profit is

$$\pi_T(Q) = -cQ + \int_0^Q xp f(x) dx + \int_Q^{\infty} Qp f(x) dx.$$

Timing

- First a return contract is signed by the manufacturer and retailer.
  - We do not specify how the contractual terms are determined.
- Then the retailer places an order.
- The manufacturer produces and ships products to the retailer.
- The sales season starts, the demand is realized, and the allowed unsold products (if any) are returned to the manufacturer.

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Incentives in Decentralized Systems
Lecture 3.3: Return Contracts: Analysis and Insights

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Road map

- **Analysis**.
- Insights.
- Remarks.

System-optimal inventory level

- The expected system profit is

$$\pi_T(Q) = -cQ + \int_0^Q xpf(x)dx + \int_Q^\infty Qpf(x)dx.$$

- The system optimal inventory level Q_T^* satisfies the equation

$$F(Q_T^*) = 1 - \frac{c}{p}.$$

- We hope that there is a return contract (w, r, R) that makes the retailer order Q_T^*.

Retailer's ordering strategy

- Under the return contract, the retailer's expected profit is

$$\pi_R(Q) = -Qw + \int_0^{(1-R)Q} (xp + RQr) f(x) dx + \int_{(1-R)Q}^{Q} \Big[xp + (Q-x)r \Big] f(x) dx + \int_Q^{\infty} Qpf(x) dx.$$

- Let's differentiate it... How?!?!?!
- We need the Leibniz integral rule: Suppose $f(x, y)$ is a function such that $\frac{\partial}{\partial y} f(x, y)$ exists and is continuous, then we have

$$\frac{d}{dy} \int_{a(y)}^{b(y)} f(x, y) dx = f(b(y), y) b'(y) - f(a(y), y) a'(y) + \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x, y) dx$$

Retailer's ordering strategy

- Let's apply the Leibniz integral rule

$$\frac{d}{dy}\int_{a(y)}^{b(y)} f(x,y)dx = f(b(y),y)b'(y) - f(a(y),y)a'(y) + \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y)dx$$

to the retailer's expected profit function $\pi_R(Q)$:

Inside $\pi_R(Q)$	Inside $\pi'_R(Q)$
$-Qw$	$-w$
$\int_0^{(1-R)Q} (xp + RQr) f(x)dx$	$(1-R)\Big[(1-R)Qp + RQr\Big] f\Big((1-R)Q\Big)$ $+ \int_0^{(1-R)Q} Rrf(x)dx$
$\int_{(1-R)Q}^{Q} \Big[xp + (Q-x)r\Big] f(x)dx$	$Qpf(Q)$ $-(1-R)\Big[(1-R)Qp - RQr\Big] f\Big((1-R)Q\Big)$ $+ \int_{(1-R)Q}^{Q} rf(x)dx$
$\int_Q^{\infty} Qpf(x)dx$	$-Qpf(Q) + \int_Q^{\infty} pf(x)dx$

Retailer's ordering strategy

- We then have

$$\pi'_R(Q) = -w + \int_0^{(1-R)Q} Rrf(x)dx + \int_{(1-R)Q}^{Q} rf(x)dx + \int_Q^{\infty} pf(x)dx$$
$$= w + RrF\Big((1-R)Q\Big) + r\Big[F(Q) - F\Big((1-R)Q\Big)\Big] + p\Big[1 - F(Q)\Big]$$
$$= -w + p - (p-r)F(Q) - (1-R)rF\Big((1-R)Q\Big).$$

- Given (w, r, R), the retailer may numerically search for Q_R^* that satisfies $\pi'_R(Q_R^*) = 0$. This is the retailer's ordering strategy.
  - Why $\pi'_R(Q) = 0$ always has a unique root?

Inducing the system-optimal inventory level

- The system-optimal inventory level Q_T^* satisfies

$$F(Q_T^*) = 1 - \frac{c}{p} = \frac{p-c}{p}.$$

- To induce the retailer to order Q_T^*, we must make Q_T^* optimal for the retailer. Therefore, we need $\pi'_R(Q_T^*) = 0$, i.e.,

$$\begin{aligned}\pi'_R(Q_T^*) &= -w + p - (p-r)F(Q_T^*) - (1-R)rF\Big((1-R)Q_T^*\Big) \\ &= -w + p - \frac{(p-c)(p-r)}{p} - (1-R)rF\Big((1-R)Q_T^*\Big) = 0.\end{aligned}$$

- To achieve coordination, we need to choose (w, r, R) to make the above equation hold, where Q_T^* is uniquely determined by $F(Q_T^*) = \frac{p-c}{p}$.
- Is it possible?

Road map

- Analysis.
- **Insights**.
- Remarks.

Extreme 1: full return with full credit

$$\pi'_R(Q_T^*) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q_T^*\Big).$$

- Let's consider the most generous return contract.

Proposition 1

If $r = w$ and $R = 1$, $\pi'_R(Q_T^*) = 0$ if and only if $c = 0$.

Proof. If $r = w$ and $R = 1$, $\pi'_R(Q_T^*) = 0$ becomes

$$w - p + \frac{(p-c)(p-w)}{p} = (p-w)\left(\frac{p-c}{p} - 1\right) = 0.$$

As $p > w$, we need $\frac{p-c}{p} = 1$, i.e., $c = 0$. □

- Allowing full returns with full credits is generally system suboptimal.

Extreme 2: no return

$$\pi'_R(Q_T^*) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q_T^*\Big).$$

- Let's consider the least generous return contract.

Proposition 2

If $r = 0$ or $R = 0$, $\pi'_R(Q_T^*) = 0$ is impossible.

Proof. If $r = 0$, $\pi'_R(Q_T^*) = 0$ becomes $w - c = 0$, which cannot be true. If $R = 0$, it becomes

$$w - c + \frac{(p-c)(p-r)}{p} + rF(Q_T^*) = w - c = 0,$$

which is again impossible. □

- Allowing no return is system suboptimal.

Full returns with partial credits

$$\pi'_R(Q_T^*) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q_T^*\Big).$$

- Let's consider full returns with partial credits.

Proposition 3

- *If* $R = 1$, $\pi'_R(Q_T^*) = 0$ *if and only if* $w = p - \frac{(p-c)(p-r)}{p}$.
- *For any* p *and* c, *a pair of* r *and* w *such that* $0 < r < w$ *can always be found to satisfy the above equation.*

Proof. When $R = 1$, the first part is immediate. According to the equation, we need $r = \frac{p(w-c)}{p-c}$. Then $w < p$ implies $\frac{p(w-c)}{p-c} < w$ and $c < w$ implies $\frac{p(w-c)}{p-c} > 0$. □

- Allowing full returns with partial credits can be system optimal!
- In this case, we say the return contract **coordinates** the system.

Profit splitting

- Under a full return contract, channel coordination requires

$$w = p - \frac{(p-c)(p-r)}{p} = c + \left(\frac{p-c}{p}\right)r.$$

  - The expected system profit is maximized. The "pie" is maximized.
  - Is this pie split **fairly** under a coordinating return contract?
  - As fairness means differently in different scenarios, we hope the pie can be split **arbitrarily**.
- In one limiting case (though not possible), when $w = c$, we need $r = 0$. In this case, $\pi_M^* = 0$ and $\pi_R^* = \pi_T^*$.
- In another limiting case, when $w = p$, we need $r = p$. In this case, $\pi_M^* = \pi_T^*$ and $\pi_R^* = 0$.
- How about the intermediate cases?

Profit splitting

- Let's visualize the set of coordinating full return contracts:

- As $\pi_M(\cdot)$ is continuous in w and r, π_M^* must **gradually** go up from 0 to π_T^* as w goes from c to p.
  - Though we did not prove that π_M^* is nondecreasing in w, it is not needed.
  - π_R^* must gradually do down as w goes from p to c.
  - **Arbitrary profit splitting** can be done!

Coordination and win-win

- We know that return contracts can be **coordinating**.
- Now we know they can also be **win-win**.
  - We can make the pie the largest.
  - We can split the pie in any way we want.
  - We can always make both players happy.
- The two players will **agree** to adopt a coordinating return contract.
- Consumers also benefit from channel coordination. Why?
- Not all coordinating contracts are win-win.

Road map

- Analysis.
- Insights.
- **Remarks**.

More in the paper

- We only introduced the main idea of the paper.
- There are still a lot untouched:
  - Salvage values and shortage costs.
  - Monotonicity of the manufacturer's and retailer's expected profit.
  - Environments with multiple retailers.
- You are encouraged, though not required, to read the paper.

Channel or supply chain coordination

- A hot topic in 1980's and 1990's.
- Not so hot now.
- Other contracts to coordinate a channel or a supply chains:
  - Two-part tariffs.
  - Quantity flexible contracts.
  - Revenue-sharing contracts.
  - Options.