IM 7011: Information Economics

Lecture 6: Adverse Selection: Screening Introduction and the Two-Type Model

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Road map

- **Introduction to screening**.
- First best with complete information.
- Incentives and the revelation principle.
- Finding the second best.

Principal-agent model

- Our introduction of **information asymmetry** will start here.
- We will study various kinds of **principal-agent** relationships.
- In the model, there is one **principal** and one or multiple **agents**.
  - The principal is the one that designs a mechanism/contract.
  - The agents act according to the mechanism/contract.
  - They are mechanism/contract **designers** and **followers**, respectively.
- It is also possible to have multiple principals competing for a single agent by offering mechanisms. This is the **common agency** problem.
- We will focus on problems with one principal and one agent.

Asymmetric information

- There are two kinds of asymmetric information:
  - **Hidden information**, which causes the **adverse selection** problem.
  - **Hidden actions**, which cause the **moral hazard** problem.
- For adverse selection, there are two basic problems:
  - When the agent has private information, the principal faces a **screening** problem.
  - When the principal has private information, the principal faces a **signaling** problem.
- Our plan in this course:

Topic	Number of weeks	Chapter
Adverse selection: screening	4	2
Moral hazard	2	4
Multi-dimensional problems	2	6
Adverse selection: signaling	3	3

- There are much more...

Adverse selection: screening

- In this lecture, we start our introduction to the **screening** problem.
- Consider the following buyer-seller relationship:
  - A manufacturer decides to buy a critical component of its product.
  - She finds a supplier that supplies this part.
  - Two machines can make this part with **different unit costs**.
  - When a manufacturer faces the supplier, she **does not know** which machine is owned by the supplier.
  - How much should the manufacturer pay for the part?
- The difficulty is:
  - If I know the supplier's cost is low, I will be able to ask for a low price.
  - However, if I ask him, he will **always** claim that his cost is high!
- The manufacturer wants to find a way to **screen** the supplier's **type**.

Adverse selection: screening

- An agent always want to **hide his type** to get bargaining power!
  - The "type" of an agent is a part of his **utility function** that is **private**.
  - We will start from **one-dimensional** problems: The agent only possesses one piece of private information.
- In the previous example:
  - The manufacturer is the principal.
  - The supplier is the agent.
  - The unit production cost is the agent's type.
- More examples:
  - A retailer does not know how to charge an incoming consumer because the consumer's **willingness-to-pay** is hidden.
  - An adviser does not know how to assign reading assignments to her graduate students because the students' **reading ability** is hidden.

Mechanism design

- One way to deal with agents' private information is to become more knowledgeable.
  - What if this is impossible or too costly?
- The standard way to screen a type is through **mechanism design**.
  - Or in the business world, **contract design**.
  - The principal will design a mechanism/contract that can "find" the agent's type.
- In the following four lectures, we will introduce the main concepts of mechanism design to you.
- We will start from the easiest case: The agent's type has only two possible values. In this case, there are **two types** of agents.

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Monopoly pricing

- We will use a **monopoly pricing** problem to illustrate the ideas.
- Imagine that you produce and sell one product.
- You are the only one who are able to produce and sell this product.
- How would you price your product to maximize your profit?

Monopoly pricing

- Suppose the demand function is $q(p) = 1 - p$. You will solve

$$\pi^* = \max\ (1-p)p \quad \Rightarrow \quad p^* = \frac{1}{2} \quad \Rightarrow \quad \pi^* = \frac{1}{4}.$$

- Note that such a demand function means consumers' **valuation** (willingness-to-pay) lie uniformly within $[0, 1]$.
  - A consumer's utility is $v - p$, where v is his valuation.
- We may visualize the **producer surplus** and **consumer surplus**:

Monopoly pricing

- Here comes a critic:
  - “Some people are willing to pay more, but your price is too low!”
  - “Some potential sales are lost because your price is too high!”
- His (useless) suggestion is:
  - “Who told you that you may set only one price?”
  - “Ask them how they like the product and charge differently!”
- Does that work?
- **Price discrimination** is impossible if consumers’ valuations are completely hidden to you.
- If you can see the valuation, you will charge each consumer his valuation. This is **perfect price discrimination**.

Information asymmetry and inefficiency

- Let's visualize the consumer and producer surpluses under perfect price discrimination:

- Information asymmetry causes **inefficiency** and reduces **social welfare** (sum of consumer and producer surpluses).
- Note that decentralization does not necessarily cause inefficiency. Here information asymmetry is the reason!

Hidden valuation and price discrimination

- In general, no consumer would be willing to tell you his preference.
  - Otherwise his surplus will be zero.
- As a producer, you try to **discriminate** (differentiate) consumers based on some special attributes/characteristics.
  - Fees for events are different for students, adults, seniors, etc.
  - Orbitz.com suggests more expensive hotels to you if you connect to their website through a Mac.[1]
- Sellers are guessing how much one is willing to pay.
  - They are not trying to be nice to students like you!
  - At least you agree that they are not trying to be nice to PC users.

[1] Google "On Orbitz, Mac Users Steered to Pricier Hotels" and read the article on the Wall Street Journal.

The two-type model

- Let's start our analysis.
- Consider the easiest case with valuation heterogeneity: There are **two** kinds of consumers.
- When obtaining q units by paying T, a **type-θ** consumer's utility is

$$u(q, T, \theta) = \theta v(q) - T.$$

  - $\theta \in \{\theta_L, \theta_H\}$ where $\theta_L < \theta_H$. θ is the consumer's **private** information.
  - $v(q)$ is strictly increasing and strictly concave. $v(0) = 0$.
  - A **high-type** (**type-H**) consumer's θ is θ_H.
  - A **low-type** (**type-L**) consumer's θ is θ_L.
  - The seller believes that $\Pr(\theta = \theta_L) = \beta = 1 - \Pr(\theta = \theta_H)$.
- The unit production cost of the seller is c. $c < \theta_L$.
- By selling q units and receiving T, the seller earns $T - cq$.
- How would you price your product to maximize your expected profit?

The two-type model with complete information

- Under complete information, the seller sees the consumer's type.
- Facing a type-H consumer, the seller solves

$$\begin{aligned} \max_{q_H \geq 0, T_H \text{ urs.}} \quad & T_H - cq_H \\ \text{s.t.} \quad & \theta_H v(q_H) - T_H \geq 0. \end{aligned}$$

- To solve this problem, note that the constraint must be **binding** (i.e., being an equality) at any optimal solution.
  - Otherwise we will increase T_H.
  - Any optimal solution satisfies $\theta_H v(q_H) - T_H = 0$.
  - The problem is equivalent to

$$\max_{q_H \geq 0} \; \theta_H v(q_H) - cq_H.$$

  - The FOC characterize the optimal quantity $\tilde{q}_H$: $\theta_H v'(\tilde{q}_H) = c$.
  - The optimal transfer is $\tilde{T}_H = \theta_H v(\tilde{q}_H)$.

The two-type model with complete information

- For the type-i consumer, the **first-best** solution $(\tilde{q}_i, \tilde{T}_i)$ satisfies

$$\theta_i v'(\tilde{q}_i) = c \quad \text{and} \quad \tilde{T}_i = \theta_i v(\tilde{q}_i) \quad \forall i \in \{L, U\}$$

- The **rent** of the consumer is his surplus of trading.
- In either case, he receives **no rent**!
- The seller extracts all the rents from the consumer.
- Next we will introduce the optimal pricing plan under information asymmetry and, of course, deliver some insights to you.

Road map

- Introduction to screening.
- First best with complete information.
- **Incentives and the revelation principle**.
- Finding the second best.

Pricing under information asymmetry

- When the valuation is hidden, the first-best pricing plan does not work.
  - You cannot make an offer (a pair of q and T) according to his type.
- How about offering a **menu** of two contracts, $\{(\tilde{q}_L, \tilde{T}_L), (\tilde{q}_H, \tilde{T}_H)\}$, for the consumer to select?
- You **cannot expect** the type-i consumer to select $(\tilde{q}_i, \tilde{T}_i)$, $i \in \{L, H\}$!
  - Both types will select $(\tilde{q}_L, \tilde{T}_L)$.
  - In particular, the type-H consumer will earn a **positive rent**:

$$\begin{aligned} u(\tilde{q}_L, \tilde{T}_L, \theta_H) &= \theta_H v(\tilde{q}_L) - \tilde{T}_L \\ &= \theta_H v(\tilde{q}_L) - \theta_L v(\tilde{q}_L) \\ &= (\theta_H - \theta_L) v(\tilde{q}_L) > 0. \end{aligned}$$

- It turns out that the first-best solution is not optimal under information asymmetry.

Incentive compatibility

- The first-best menu $\{(\tilde{q}_L, \tilde{T}_L), (\tilde{q}_H, \tilde{T}_H)\}$ is said to be **incentive-incompatible**:
  - The type-H consumer has an incentive to **hide** his type and **pretend** to be a type-L one.
  - This fits our common intuition!
- A menu is **incentive-compatible** if different types of consumers will select different contracts.
  - An incentive-compatible contract induces **truth-telling**.
  - According to his selection, we can identify his type!
- How to make a menu incentive-compatible?

Incentive-compatible menu

- Suppose a menu $\{(q_L, T_L), (q_H, T_H)\}$ is incentive-compatible.
  - The type-H consumer will select (q_H, T_H), i.e.,

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L.$$

  - The type-L consumer will select (q_L, T_L), i.e.,

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H.$$

- The above two constraints are called the **incentive-compatibility constraints** (IC constraints) or **truth-telling** constraints.
- If the seller wants to do business with both types, she also needs the **individual-rationality constraints** (IR constraints) or **participation** constraints:

$$\theta_i v(q_i) - T_i \geq 0 \quad \forall i \in \{L, H\}.$$

- The seller may offer an incentive-compatible menu. But is it optimal?

Optimal pricing under information asymmetry

- In general, we are determining a pricing scheme, i.e., a function $T(q)$ that maps a quantity to a price.
  - A **linear pricing scheme** satisfies $T(q) = pq$ for some $p \in \mathbb{R}$.
  - A **single two-part tariff** satisfies $T(q) = pq + Z$ for some $p, Z \in \mathbb{R}$.
- A menu $\{(q_L, T_L), (q_H, T_H)\}$ is

$$T(q) = \begin{cases} T_L & \text{if } q = q_L \\ T_H & \text{if } q = q_H \\ \infty & \text{otherwise} \end{cases} .$$

Optimal pricing under information asymmetry

- Assume that we are restricted to the above to pricing schemes, we may formulate the problems and solve them.
- Even if we do so, we cannot guarantee that any of them is optimal.
  - Because we did not investigate other pricing schemes!
- Let's consider the **most general** pricing problem.

Optimal pricing under information asymmetry

- Our decision "variables" is now a function of quantity: $T(q)$.
  - This is a menu of **infinite** contracts.
- Being offered the function, how does the type-i consumer response?
  - He tries to maximize his utility $u(\theta_i, q, T(q)) = \theta_i v(q) - T(q)$.
  - His purchasing quantity, $\hat{q}_i$, satisfies[2]

$$\hat{q}_i \in \operatorname*{argmax}_q \Big\{ \theta_i v(q) - T(q) \Big\}.$$

  - He will really buy something if and only if

$$\theta_i v(\hat{q}_i) - T(\hat{q}_i) \geq 0.$$

- With the consumer's response, in expectation the seller earns

$$\beta \Big[T(\hat{q}_L) - c\hat{q}_L \Big] + (1 - \beta) \Big[T(\hat{q}_H) - c\hat{q}_H \Big].$$

[2] It is false to write $\theta_i v'(\hat{q}_i) - T'(\hat{q}_i) = 0$. Why?

Optimal pricing under information asymmetry

- The complete formulation is

$$\begin{aligned} \max_{T(\cdot)} \quad & \beta\Big[T(\hat{q}_L) - c\hat{q}_L\Big] + (1-\beta)\Big[T(\hat{q}_H) - c\hat{q}_H\Big] \\ \text{s.t.} \quad & \hat{q}_i \in \operatorname*{argmax}_q \Big\{\theta_i v(q) - T(q)\Big\} \quad \forall i \in \{L, H\} \\ & \theta_i v(\hat{q}_i) - T(\hat{q}_i) \geq 0 \quad \forall i \in \{L, H\}. \end{aligned}$$

- Having a formulation is good. But how to solve it?
- It is impossible to solve it!
- Unless we apply the **revelation principle**.

Revelation principle

- Among all possible pricing schemes (or mechanisms, in general), some are incentive compatible while some are not.
  - A single contract is not.
  - The first-best menu is not.
  - An incentive compatible menu is.
- The revelation principle tells us "Among all **incentive compatible** mechanisms, at least **one is optimal**."
  - We may restrict our attentions to incentive-compatible menus!
  - The problem then becomes tractable.
- Contributors of the revelation principle include three Nobel Laureates: James Mirrlees in 1996, and Eric Maskin and Roger Myerson in 2007.
  - There are other contributors.
  - Related works were published in 1970s.
- Let's discuss why it is true.

The idea of the revelation principle

- In general, the principal designs a mechanism for the agent(s).
  - The mechanism specifies a game rule. Agents act according to the rules.
- When agents have private types, there are two kinds of mechanisms.
- Under an **indirect mechanism**:
  - The principal specifies a function mapping agents' actions to payoffs.
  - Each agent, based on his type and his belief on other agents' types, acts to maximize his expected utilities.
- Under a **direct mechanism**:
  - The principal specifies a function mapping agents' **reported types** to actions and payoffs.
  - Each agent, based on his type and his belief on other agents' types, **reports a type** to maximize his expected utilities.
- If a direct mechanism can reveal agents' types (i.e., making all agents report truthfully), it is a **direct revelation mechanism**.

The idea of the revelation principle

Proposition 1 (Revelation principle)

Given any equilibrium of any given indirect mechanism, there is a direct revelation mechanism under which the equilibrium is equivalent to the given one: In the two equilibria, agents do the same actions.

- The idea is to "imitate" the given equilibrium.
- The given equilibrium specifies each agent's (1) strategy to map his type to an action and (2) his expected payoff.
- We may "construct" a direct mechanism as follows:
  - Given any type report (some types may be false), find the **corresponding actions and payoffs** in the given equilibrium as if the agents' types are really as reported.
  - Then assign **exactly those actions and payoffs** to agents.
- If the agents all report truthfully under the direct mechanism, they are receiving exactly what they receive in the given equilibrium. Therefore, under the direct mechanism no one deviates.

Reducing the search space

- How to simplify our pricing problem with the revelation principle?
  - We only need to search among menus that can induce truth-telling.
  - Different types of consumers should select different contracts.
  - As we have only two consumers, two contracts are sufficient.
  - One is not enough and three is too many!
- The problem to solve is

$$
\begin{aligned}
\max_{q_H, T_H, q_L, T_L} \quad & \beta\Big[T_L - cq_L\Big] + (1-\beta)\Big[T_H - cq_H\Big] && \text{(OBJ)} \\
\text{s.t.} \quad & \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\
& \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H && \text{(IC-L)} \\
& \theta_H v(q_H) - T_H \geq 0 && \text{(IR-H)} \\
& \theta_L v(q_L) - T_L \geq 0. && \text{(IR-L)}
\end{aligned}
$$

  - The two IC constraints ensure truth-telling.
  - The two IR constraints ensure participation.
- Next we will introduce how to solve this problem.

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Solving the two-type problem

- Below we will introduce the standard way of solving the standard two-type problem.
- The key is that we want to **analytically** solve the problem.
  - With the analytical solution, we may generate some insights.

Step 1: Monotonicity

- By adding the two IC constraints

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L$$

and

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H,$$

we obtain

$$\begin{aligned} & \theta_H v(q_H) + \theta_L v(q_L) \geq \theta_H v(q_L) + \theta_L v(q_H) \\ \Rightarrow\ & (\theta_H - \theta_L) v(q_H) \geq (\theta_H - \theta_L) v(q_L) \\ \Rightarrow\ & v(q_H) \geq v(q_L) \\ \Rightarrow\ & q_H \geq q_L. \end{aligned}$$

- This is the **monotoniciy** condition: In an incentive-compatible menu, the high-type consumer consume more.
  - Intuition: The high-type consumer prefers a high consumption.

Step 2: (IR-H) is redundant

- (IC-H) and (IR-L) imply that (IR-H) is redundant:

$$\begin{aligned} \theta_H v(q_H) - T_H &\geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\ &> \theta_L v(q_L) - T_L && (\theta_H > \theta_L) \\ &\geq 0. && \text{(IR-L)} \end{aligned}$$

- The high-type consumer earns **a positive rent**. Full surplus extraction is impossible under information asymmetry.
- The problem reduces to

$$\begin{aligned} \max_{q_H, T_H, q_L, T_L} \quad & \beta\Big[T_L - cq_L\Big] + (1-\beta)\Big[T_H - cq_H\Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\ & \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H && \text{(IC-L)} \\ & \theta_L v(q_L) - T_L \geq 0. && \text{(IR-L)} \end{aligned}$$

Step 3: Ignore (IC-L)

- Let's "guess" that (IC-L) will be redundant and ignore it for a while.
- We make this guess because the low-valuation consumer **has no incentive** to pretend to have a high valuation.
- We will eventually verify that the optimal solution of the relaxed program indeed satisfies (IC-L).
- The problem reduces to

$$
\begin{aligned}
\max_{q_H, T_H, q_L, T_L} \quad & \beta\Big[T_L - cq_L\Big] + (1-\beta)\Big[T_H - cq_H\Big] && \text{(OBJ)} \\
\text{s.t.} \quad & \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\
& \theta_L v(q_L) - T_L \geq 0. && \text{(IR-L)}
\end{aligned}
$$

Step 4: Remaining constraints bind at optimality

$$\max_{q_H, T_H, q_L, T_L} \quad \beta\Big[T_L - cq_L\Big] + (1-\beta)\Big[T_H - cq_H\Big] \qquad \text{(OBJ)}$$

$$\text{s.t.} \quad \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \qquad \text{(IC-H)}$$

$$\theta_L v(q_L) - T_L \geq 0. \qquad \text{(IR-L)}$$

- (IC-H) must be **binding** at any optimal solution:
  - The seller wants to increase T_H as much as possible.
  - She will keep doing so until (IC-H) is binding.
- (IR-L) must also be **binding** at any optimal solution:
  - The seller wants to increase T_L as much as possible.
  - She will keep doing so until (IR-L) is binding.
  - Note that increasing T_L makes (IC-H) more relaxed rather than tighter.
- Note that if we did not ignore (IC-L), i.e.,

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H,$$

then we cannot claim that (IR-L) is binding!

Step 5: Removing the transfers

- The problem reduces to

$$\max_{q_H, T_H, q_L, T_L} \quad \beta\Big[T_L - cq_L\Big] + (1-\beta)\Big[T_H - cq_H\Big] \qquad \text{(OBJ)}$$

$$\text{s.t.} \quad \theta_H v(q_H) - T_H = \theta_H v(q_L) - T_L \qquad \text{(IC-H)}$$

$$\theta_L v(q_L) - T_L = 0. \qquad \text{(IR-L)}$$

- Therefore, we may remove the two constraints and replace T_L and T_H in (OBJ) by $\theta_L v(q_L)$ and $\theta_H v(q_H) - \theta_H v(q_L) + \theta_L v(q_L)$, respectively.
- The problem reduces to an **unconstrained** problem

$$\max_{q_H, q_L} \quad \beta\Big[\theta_L v(q_L) - cq_L\Big] + (1-\beta)\Big[\theta_H v(q_H) - \theta_H v(q_L) + \theta_L v(q_L) - cq_H\Big].$$

Step 6: Solving the unconstrained problem

$$\max_{q_H, q_L} \; \beta\Big[\theta_L v(q_L) - cq_L\Big] + (1-\beta)\Big[\theta_H v(q_H) - cq_H - (\theta_H - \theta_L)v(q_L)\Big].$$

- Because $v(\cdot)$ is strictly concave, the reduced objective function is strictly concave in q_H and q_L.
- If $\frac{\theta_H - \theta_L}{\theta_H} < \beta$, the **second-best** solution $\{(q_L^*, T_L^*), (q_H^*, T_H^*)\}$ satisfies[3]

$$\theta_H v'(q_H^*) = c \quad \text{and} \quad \theta_L v'(q_L^*) = c\left[\frac{1}{1 - (\frac{1-\beta}{\beta}\frac{\theta_H - \theta_L}{\theta_L})}\right].$$

[3] If $\frac{\theta_H - \theta_L}{\theta_H} \geq \beta$, $q_L^* = 0$ and q_H^* satisfies $\theta_H v'(q_H^*) = c$.

Step 7: Verifying that (IC-L) is satisfied

- To verify that (IC-L) is satisfied, we apply

$$T_L = \theta_L v(q_L) \quad \text{and} \quad T_H = \theta_H v(q_H) - (\theta_H - \theta_L) v(q_L).$$

- With this, (IC-L)

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H$$

is equivalent to

$$0 \geq -(\theta_H - \theta_L)\Big[v(q_H) + v(q_L)\Big].$$

Therefore, (IC-L) is satisfied.

Inefficient consumption levels

- Recall that the first-best consumption levels $\tilde{q}_L$ and $\tilde{q}_H$ satisfies

$$\theta_H v'(\tilde{q}_H) = c \quad \text{and} \quad \theta_L v'(\tilde{q}_L) = c.$$

  Moreover, the second-best consumption levels

$$\theta_H v'(q_H^*) = c \quad \text{and} \quad \theta_L v'(q_L^*) = c\left[\frac{1}{1 - (\frac{1-\beta}{\beta}\frac{\theta_H - \theta_L}{\theta_L})}\right] > c.$$

- The high-type consumer consumes the **first-best** amount.
- For the low-type consumer, $v'(\tilde{q}_L) = \frac{c}{\theta_L} < v'(q_L^*)$. As $v(\cdot)$ is strictly concave, $q_L^* < \tilde{q}_L$.
- The low-type consumer consumes **less** than the first-best amount.
  - Information asymmetry causes inefficiency.
  - The consumption will only decrease. It will not become larger. Why?

Cost of inducing truth-telling

- Regarding the consumption levels:
  - We have $q_L^* < \tilde{q}_L$. Why do we decrease q_L?
  - Recall that under the first-best menu, the high-type consumer pretends to have a low valuation.
  - Because he prefers a high consumption level, we must **cut down** q_L to make him **unwilling to lie**.
  - Inevitably, decreasing q_L creates inefficiency.
- Regarding the consumer surplus:
  - In equilibrium, the low-type consumer earns $\theta_L v(q_L^*) - T_L^* = 0$.
  - However, the high-type consumer earns

$$\theta_H v(q_H^*) - T_H^* = (\theta_H - \theta_L) v(q_L^*) > 0.$$

  - The high-type consumer earns a positive **information rent**.
  - One with his type hidden can earn a positive rent **in expectation**.
- Note that the high-type consumer's rent depends on q_L^*.
- Cutting down q_L^* is to cut down his information rent!

Summary

- We discussed a two-type monopoly pricing problem.
- We found the first-best and second-best mechanisms.
  - Thanks to the revelation principle!
- We found that information asymmetry brings in inefficiency.
- For the second-best solution:
  - **Monotonicity**: The high-type consumption level is higher.
  - **Efficiency at top**: The high-type consumption level is efficient.
  - **No rent at bottom**: The low-type consumer earns no rent.
- Information asymmetry **protects the agent**.
  - But it hurts the principal and social welfare.