Information Economics, Fall 2015
Midterm Exam

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Note. This exam is in-class and open everything (including all kinds of electronic devices). However, an exam taker is not allowed to communicate with any person during the exam. Cheating will result in severe penalty. You do not need to return the problem sheet. The maximum number of points that one may earn is 100.

1. (30 points) A manufacturer sells to a newsvendor retailer. The market demand is uniform between 0 and 1 and the unit retail price is 1. To produce x units, the manufacturer must pay $\frac{1}{2}cx^2$ as the production cost. The sequence of events are: (1) the manufacturer announces a wholesale price w, (2) the retailer orders q units, and then (3) the manufacturer decides her production quantity Q and sell Q units to the retailer. Note that because the manufacturer's marginal production cost increases as the production quantity increases, there is really a reason to see Q to be strictly less than q. Nevertheless, the manufacturer cannot sell more than q units to the retailer. In other words, the manufacturer is subject to a constraint $Q \leq q$. There is no penalty for partial fulfillment. Also note that the announced wholesale price must be followed under full or partial fulfillment. Each firm acts to maximize her or his own expected profit.

   (a) (5 points) Given the announced wholesale price w and the order quantity q, find the manufacturer's optimal production quantity Q^* as a function of w and q.

   (b) (5 points) Given the announced wholesale price w, find the retailer's optimal order quantity q^* as a function of w.

   (c) (5 points) Find the manufacturer's optimal wholesale price w^* as a function of c.

   (d) (10 points) Suppose now the manufacturer may offer a return contract (w, r). Under a return contract (w, r), the retailer pays the wholesale price w to buy products and may return all the unsold products to the manufacturer and receive the the return credit r per unit returned. All other model settings remain the same. Without any mathematical derivation, intuitively explain whether the manufacturer will offer a positive return credit in equilibrium.

2. (50 points) Two manufacturers and one retailer exist in a market. Each of the two manufacturers may decide whether to sell to end consumers directly or sell through the retailer. They make the channel selection decision simultaneously. If manufacturer i sells to consumers directly, she should choose a retail price p_i; otherwise, she should choose a wholesale price w_i. If manufacturer i sells through the retailer, the retailer will choose a retail price p_i. Note that p_1 and p_2 may be different if both manufacturers sell through the retailer.

   Obviously, there are in total four market channel structures: II, ID, DI, and DD, where I means "indirect" (sells through the retailer) and D means "direct" (sells to consumers directly). In DD, the two manufacturers choose their retail prices p_is simultaneously. In II, first the two manufacturers choose their wholesale prices w_is simultaneously, and then the retailer chooses the retail prices p_is. In ID, manufacturer 1 chooses her wholesale price w_1, and then manufacturer 2 and the retailer chooses their retail prices p_2 and p_1 simultaneously. DI is the opposite of ID.

   In any case, consumers will observe two retail prices p_1 and p_2, one for the products made by one manufacturer. The demand for the two products will be

   $$q_1 = 1 - p_1 + \theta p_2 \text{ and } q_2 = 1 - p_2 + \theta p_1,$$

   where $\theta \in [0, 1)$ measures competition intensity: Higher θ means more intense competition. Each firm acts to maximize her or his own profit. The production cost is normalized to 0.

(a) (5 points) Suppose all the three firms integrate to make their decisions. Find the optimal retail prices for the "integrated firm."

(b) (5 points) In DD, find the equilibrium retail prices.

(c) (5 points) In II, find the retailer's optimal retail prices given wholesale prices w_1 and w_2. Treat w_1 and w_2 as exogenous.

(d) (5 points) Continue from Part (c). In II, find the manufacturers' equilibrium wholesale prices.

(e) (5 points) In ID, find all the equilibrium prices.

(f) (5 points) Let's compare pure integration with II. Intuitively, due to double marginalization, the equilibrium retail prices should be higher under II than under pure integration. Prove or disprove this intuition.

(g) (10 points) Let's compare DD with II. Intuitively, due to double marginalization, the equilibrium retail prices should be higher under II than under DD. Prove or disprove this intuition.

(h) (10 points) Let's compare DD with ID. Intuitively, due to double marginalization, the equilibrium retail price of product 1 should be higher under ID than under DD. Therefore, manufacturer 2 has a larger room to charge a higher price. The retail price of product 2 should therefore also become higher under ID than under DD. Prove or disprove this intuition.

3. (50 points) A retailer hires a salesperson to sell to end consumers. The market condition θ may be either θ_L or θ_H, where $\Pr(\theta = \theta_L) = \frac{1}{2} = \Pr(\theta = \theta_H)$. The salesperson can choose his effort level a by paying $\frac{1}{2}a^2$ as the cost of effort exertion. The market demand $x \in \{0, 1\}$, where $\Pr(x = 1) = \theta a = 1 - \Pr(x = 0)$. The salesperson may forecast to obtain a demand signal $s \in \{s_B, s_G\}$. We have

$$\lambda = \Pr(s = s_B | \theta = \theta_L) = \Pr(s = s_G | \theta = \theta_H)$$

be defined as the salesperson's exogenous forecasting accuracy. The retailer has no signal. If the two players agree with a contract (u, v), the salesperson's salary will be $u + vx$. The sequence of events are (1) the retailer offers a menu of contracts $((u_G, v_G), (u_B, v_B))$, (2) the salesperson selects a contract, (3) the salesperson chooses his effort level. Each of the two players acts to maximize her or his expected profit.

(a) (5 points) Let $P_{ij} = \Pr(\theta = \theta_i | s = s_j)$, $i \in \{H, L\}$, $j \in \{G, B\}$. Find P_{LB} as a function of λ.

(b) (5 points) Let $N_j = \mathbb{E}[\theta | s = s_j]$, $j \in \{s_G, s_B\}$. Find N_B as a function of θ_L, θ_H, and λ.

(c) (5 points) Let $Q_j = \Pr(s = s_j)$. Find Q_B.

(d) (5 points) Suppose that the salesperson has observed a signal s_j and selected a contract (u_k, v_k), find his optimal effort level a^*_{jk}. You may use N_j, P_{ij}, and Q_j in your answer.

(e) (5 points) Formulate the objective function in the retailer's contract design problem. You may use N_j, P_{ij}, and Q_j in your answer.

(f) (5 points) Formulate the constraints in the retailer's contract design problem. You may use N_j, P_{ij}, and Q_j in your answer.

(g) (10 points) Find the retailer's optimal menu. You may use N_j, P_{ij}, and Q_j in your answer.

(h) (10 points) Let $\bar{a} = \sum_{j \in \{G,B\}} \Pr(s = s_j) a^*_{jj}$ be the expected effort level. Characterize how $\bar{a}$ is affected by $\lambda \in [\frac{1}{2}, 1]$.