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Suggested Solution for Homework 1

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. (a) $\nabla f(x_1, x_2) = \begin{bmatrix} 10x_1^4 + 6x_1x_2 + 3 \\ 3x_1^2 - 3x_2^3 \end{bmatrix}$, $\nabla^2 f(x_1, x_2) = \begin{bmatrix} 40x_1^3 + 6x_2 & 6x_1 \\ 6x_1 & -6x_2 \end{bmatrix}$.

   (b) $\frac{d}{dx} f(x) = \frac{3x^2 + 2}{x^3 + 2x} e^{3x} + 3\ln(x^3 + 2x)e^{3x}$.

   (c) $\int f(x) dx_2 = \frac{1}{3} x_1 x_2^3 + \frac{1}{2} x_1 e^{2x_2}$.

   (d) $\frac{d}{dx} \int_0^x (t^3 + 3x - 2)dt = x^3 + 6x - 2$.

   (e) $\mathbb{E}[X] = 3.2$, $\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = 3.36$.

   (f) Because we need $\int_0^2 kx^{1.5}dx = 1$, we have $k = \frac{5}{8\sqrt{2}}$. It then follows that $\mathbb{E}[X] = \int_0^2 xf(x)dx = \int_0^2 x^{2.5} \frac{5}{8\sqrt{2}} dx = \frac{10}{7}$.

   (g) Because $\frac{d^2}{dx^2} f(x) = \frac{15}{4} x^{\frac{1}{2}} + 6 > 0$ over $[0, \infty)$, it is convex over the region.

   (h) Because $\frac{d^2}{dx^2} g(x) = -x^{-2} + 4 \geq 0$ if and only if $x \in (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$, it is convex over there.

2. (a) As shown in Figure 1, the area in gray is the feasible region. Obviously, it is not a convex set since there exists some points between point A and B that do not belong to the feasible region.

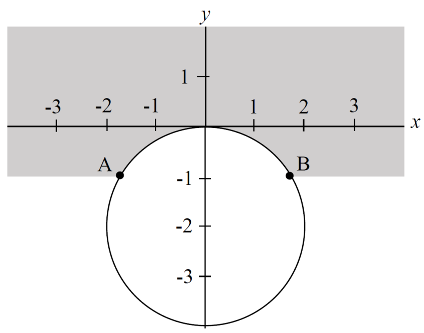

Figure 1: Graphical solution

   (b) This program is unbounded, so there is no optimal solution.

   (c) Yes, the point $(-\sqrt{3}, -1)$ is not a global maximum but a local one since there does not exist any point nearby that is greater than it.

   (d) No, there exists no point that is local maximum but not a global maximum.

3. Suppose that there are two points x and y, $x, y \in F \cap G$. Then we know $x, y \in F$ and $x, y \in G$. Since F and G are convex sets it follows that $z \in F$ and $z \in G$ where

$$z = \lambda x + (1 - \lambda)y, \lambda \in [0, 1].$$

Hence $z \in F \cap G$.

4. (a) The problem can be formulated as

$$\begin{aligned} \max_{p} \quad & \pi(p) = (p-c)a\left(1-\frac{p}{b}\right) \\ \text{s.t.} \quad & 0 \le p \le 0. \end{aligned}$$

Since $\pi''(p) = \frac{-2a}{b} < 0$ and $p \in [0, p_0]$, we maximize a concave function over a convex feasible region. Therefore, the problem is a convex program.

(b) The optimal solution is

$$p^* = \begin{cases} \frac{b+c}{2} & \text{if } \frac{b+c}{2} \le p_0 \\ p_0 & \text{otherwise} \end{cases}.$$

(c) p^* is increasing in the highest possible valuation b, because the seller can charge more when consumers' willingness to pay is higher. p^* is increasing in the unit cost c, because the seller will try to cover the additional cost by increasing the retail price. p^* has nothing to do with the total number of consumer a, because the total number of consumer will only affect the total profit but not the retail price.

5. (a) The problem can be formulated as

$$\begin{aligned} \max_{q} \quad & \pi(q) = r\mathbb{E}\Big[\min\{q, D\}\Big] + d\mathbb{E}\Big[\max\{q-x, 0\}\Big] - cq \\ \text{s.t.} \quad & q \ge 0. \end{aligned}$$

(b) First, we may rewrite $\pi(q)$ into

$$\pi(q) = r\left\{\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\right\} + d\int_0^q (q-x)f(x)dx - cq.$$

We then have

$$\begin{aligned} \pi'(q) &= r\Big\{[qf(q)] + [-qf(q) + \int_q^\infty f(x)dx]\} + d\int_0^q f(x)dx - c \\ &= r(1-F(q)) + dF(q) - c \\ &= r + (d-r)F(q) - c, \end{aligned}$$

which implies that the optimal quantity q^* satisfies

$$F(q^*) = \frac{r-c}{r-d}.$$

(c) $F(q^*)$ is increasing in r. If the unit revenue increases, the newsvendor will have an incentive to order more quantity. $F(q^*)$ is decreasing in c. If the unit cost increases, the newsvendor will choose to order less to avoid from overstocking. $F(q^*)$ is increasing in d. If the salvage value increases, the newsvendor will have an incentive to order more because the recycling site shares the risk with him.