

Information Economics

The Two-type Screening Model

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Road map

- ▶ **Introduction to screening.**
- ▶ First best with complete information.
- ▶ Incentives and the revelation principle.
- ▶ Finding the second best.
- ▶ Appendix: Proof of the revelation principle.

Principal-agent model

- ▶ Our introduction of **information asymmetry** will start here.
- ▶ We will study various kinds of **principal-agent** relationships.
- ▶ In the model, there is one **principal** and one or multiple **agents**.
 - ▶ The principal is the one that designs a mechanism/contract.
 - ▶ The agents act according to the mechanism/contract.
 - ▶ They are mechanism/contract **designers** and **followers**, respectively.
- ▶ It is also possible to have multiple principals competing for a single agent by offering mechanisms. This is the **common agency** problem.
- ▶ We will only discuss problems with one principal and one agent.

Asymmetric information

- ▶ There are two kinds of asymmetric information:
 - ▶ **Hidden information**, which causes the **adverse selection** problem.
 - ▶ **Hidden actions**, which cause the **moral hazard** problem.
- ▶ The principal may face two forms of adverse selection problems:
 - ▶ **Screening**: when the agent has private information.
 - ▶ **Signaling**: when the principal has private information.
- ▶ We have talked about the moral hazard problem.
- ▶ Today we discuss the screening problem.

Adverse selection: screening

- ▶ Consider the following buyer-seller relationship:
 - ▶ A manufacturer decides to buy a critical component of its product.
 - ▶ She finds a supplier that supplies this part.
 - ▶ Two kinds of technology can produce this component with **different unit costs**.
 - ▶ When a manufacturer faces the supplier, she **does not know** which kind of technology is owned by the supplier.
 - ▶ How much should the manufacturer pay for the part?
- ▶ The difficulty is:
 - ▶ If I know the supplier's cost is low, I will be able to ask for a low price.
 - ▶ However, if I ask him, he will **always** claim that his cost is high!
- ▶ The manufacturer wants to find a way to **screen** the supplier's **type**.

Adverse selection: screening

- ▶ An agent always want to **hide his type** to get bargaining power!
 - ▶ The “type” of an agent is a part of his **utility function** that is **private**.
- ▶ In the previous example:
 - ▶ The manufacturer is the principal.
 - ▶ The supplier is the agent.
 - ▶ The unit production cost is the agent’s type.
- ▶ More examples:
 - ▶ A retailer does not know how to charge an incoming consumer because the consumer’s **willingness-to-pay** is hidden.
 - ▶ An adviser does not know how to assign reading assignments to her graduate students because the students’ **reading ability** is hidden.

Mechanism design

- ▶ One way to deal with agents' private information is to become more knowledgeable.
- ▶ When such an information-based approach is not possible, one way to screen a type is through **mechanism design**.
 - ▶ Or in the business world, **contract design**.
 - ▶ The principal will design a mechanism/contract that can “find” the agent's type.
- ▶ We will start from the easiest case: The agent's type has only two possible values. In this case, there are **two types** of agents.

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Monopoly pricing

- ▶ We will use a **monopoly pricing** problem to illustrate the ideas.
- ▶ Imagine that you produce and sell one product.
- ▶ You are the only one who are able to produce and sell this product.
- ▶ How would you price your product to maximize your profit?

Monopoly pricing

- ▶ Suppose the demand function is $q(p) = 1 - p$. You will solve

$$\pi^* = \max (1 - p)p \quad \Rightarrow \quad p^* = \frac{1}{2} \quad \Rightarrow \quad \pi^* = \frac{1}{4}.$$

- ▶ Note that such a demand function means consumers' **valuation** (willingness-to-pay) lie uniformly within $[0, 1]$.
 - ▶ A consumer's utility is $v - p$, where v is his valuation.
- ▶ We may visualize the **monopolist's profit**:

Monopoly pricing

- ▶ Here comes a critic:
 - ▶ “Some people are willing to pay more, but your price is too low!”
 - ▶ “Some potential sales are lost because your price is too high!”
- ▶ His (useless) suggestion is:
 - ▶ “Who told you that you may set only one price?”
 - ▶ “Ask them how they like the product and charge differently!”
- ▶ Does that work?
- ▶ **Price discrimination** is impossible if consumers’ valuations are completely hidden to you.
- ▶ If you can see the valuation, you will charge each consumer his valuation. This is **perfect price discrimination**.

The two-type model

- ▶ In general, no consumer would be willing to tell you his preference.
- ▶ Consider the easiest case with valuation heterogeneity: There are **two** kinds of consumers.
- ▶ When obtaining q units by paying T , a **type- θ** consumer's utility is

$$u(q, T, \theta) = \theta v(q) - T.$$

- ▶ $\theta \in \{\theta_L, \theta_H\}$ where $\theta_L < \theta_H$. θ is the consumer's **private** information.
- ▶ $v(q)$ is strictly increasing and strictly concave. $v(0) = 0$.
- ▶ A **high-type (type-H)** consumer's θ is θ_H .
- ▶ A **low-type (type-L)** consumer's θ is θ_L .
- ▶ The seller believes that $\Pr(\theta = \theta_L) = \beta = 1 - \Pr(\theta = \theta_H)$.
- ▶ The unit production cost of the seller is c . $c < \theta_L$.
- ▶ By selling q units and receiving T , the seller earns $T - cq$.
- ▶ How would you price your product to maximize your expected profit?

The two-type model with complete information

- ▶ Under complete information, the seller sees the consumer's type.
- ▶ Facing a type-H consumer, the seller solves

$$\begin{aligned} \max_{q_H \geq 0, T_H \text{ urs.}} \quad & T_H - cq_H \\ \text{s.t.} \quad & \theta_H v(q_H) - T_H \geq 0. \end{aligned}$$

- ▶ To solve this problem, note that the constraint must be **binding** (i.e., being an equality) at any optimal solution.
 - ▶ Otherwise we will increase T_H .
 - ▶ Any optimal solution satisfies $\theta_H v(q_H) - T_H = 0$.
 - ▶ The problem is equivalent to

$$\max_{q_H \geq 0} \theta_H v(q_H) - cq_H.$$

- ▶ The FOC characterize the optimal quantity \tilde{q}_H : $\theta_H v'(\tilde{q}_H) = c$.
- ▶ The optimal transfer is $\tilde{T}_H = \theta_H v(\tilde{q}_H)$.

The two-type model with complete information

- ▶ For the type- i consumer, the **first-best** solution $(\tilde{q}_i, \tilde{T}_i)$ satisfies

$$\theta_i v'(\tilde{q}_i) = c \quad \text{and} \quad \tilde{T}_i = \theta_i v(\tilde{q}_i) \quad \forall i \in \{L, H\}$$

- ▶ The **rent** of the consumer is his surplus of trading.
- ▶ In either case, the consumer receives **no rent**!
- ▶ The seller extracts all the rents from the consumer.
- ▶ Next we will introduce the optimal pricing plan under information asymmetry and, of course, deliver some insights to you.

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Pricing under information asymmetry

- ▶ When the valuation is hidden, the first-best plan does not work.
 - ▶ You cannot make an offer (a pair of q and T) according to his type.
- ▶ How about offering a **menu** of two contracts, $\{(\tilde{q}_L, \tilde{T}_L), (\tilde{q}_H, \tilde{T}_H)\}$, for the consumer to select?
- ▶ You **cannot expect** the type- i consumer to select $(\tilde{q}_i, \tilde{T}_i)$, $i \in \{L, H\}$!
 - ▶ Both types will select $(\tilde{q}_L, \tilde{T}_L)$.
 - ▶ In particular, the type-H consumer will earn a **positive rent**:

$$\begin{aligned}u(\tilde{q}_L, \tilde{T}_L, \theta_H) &= \theta_H v(\tilde{q}_L) - \tilde{T}_L \\ &= \theta_H v(\tilde{q}_L) - \theta_L v(\tilde{q}_L) \\ &= (\theta_H - \theta_L)v(\tilde{q}_L) > 0.\end{aligned}$$

- ▶ It turns out that the first-best solution is not optimal under information asymmetry.

Incentive compatibility

- ▶ The first-best menu $\{(\tilde{q}_L, \tilde{T}_L), (\tilde{q}_H, \tilde{T}_H)\}$ is said to be **incentive-incompatible**:
 - ▶ The type-H consumer has an incentive to **hide** his type and **pretend** to be a type-L one.
 - ▶ This fits our common intuition!
- ▶ A menu is **incentive-compatible** if different types of consumers will select different contracts.
 - ▶ An incentive-compatible contract induces **truth-telling**.
 - ▶ According to his selection, we can identify his type!
- ▶ How to make a menu incentive-compatible?

Incentive-compatible menu

- ▶ Suppose a menu $\{(q_L, T_L), (q_H, T_H)\}$ is incentive-compatible.
 - ▶ The type-H consumer will select (q_H, T_H) , i.e.,

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L.$$

- ▶ The type-L consumer will select (q_L, T_L) , i.e.,

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H.$$

- ▶ The above two constraints are called the **incentive-compatibility constraints** (IC constraints) or **truth-telling** constraints.
- ▶ If the seller wants to do business with both types, she also needs the **individual-rationality constraints** (IR constraints) or **participation** constraints:

$$\theta_i v(q_i) - T_i \geq 0 \quad \forall i \in \{L, H\}.$$

- ▶ The seller may offer an incentive-compatible menu. But is it optimal?

Inducing truth-telling is optimal

- ▶ Among all possible pricing schemes (or mechanisms, in general), some are incentive compatible while some are not.
 - ▶ The first-best menu is not.
 - ▶ An incentive compatible menu is.
- ▶ The **revelation principle** tells us “Among all **incentive compatible** mechanisms, at least **one is optimal**.”¹
 - ▶ We may restrict our attentions to incentive-compatible menus!
 - ▶ The problem then becomes tractable.
- ▶ Contributors of the revelation principle include three Nobel Laureates: James Mirrlees in 1996, and Eric Maskin and Roger Myerson in 2007.
 - ▶ There are other contributors.
 - ▶ Related works were published in 1970s.

¹A nonrigorous proof is provided in the appendix.

Reducing the search space

- ▶ How to simplify our pricing problem with the revelation principle?
 - ▶ We only need to search among menus that can induce truth-telling.
 - ▶ Different types of consumers should select different contracts.
 - ▶ As we have only two consumers, two contracts are sufficient.
 - ▶ One is not enough and three is too many!
- ▶ The problem to solve is

$$\max_{q_H, T_H, q_L, T_L} \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] \quad (\text{OBJ})$$

$$\text{s.t. } \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \quad (\text{IC-H})$$

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H \quad (\text{IC-L})$$

$$\theta_H v(q_H) - T_H \geq 0 \quad (\text{IR-H})$$

$$\theta_L v(q_L) - T_L \geq 0. \quad (\text{IR-L})$$

- ▶ The two IC constraints ensure truth-telling.
- ▶ The two IR constraints ensure participation.
- ▶ Next we will introduce how to solve this problem.

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Solving the two-type problem

- Below we will introduce the standard way of solving the standard two-type problem²

$$\max_{q_H, T_H, q_L, T_L} \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] \quad (\text{OBJ})$$

$$\text{s.t.} \quad \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \quad (\text{IC-H})$$

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H \quad (\text{IC-L})$$

$$\theta_H v(q_H) - T_H \geq 0 \quad (\text{IR-H})$$

$$\theta_L v(q_L) - T_L \geq 0. \quad (\text{IR-L})$$

- The key is that we want to **analytically** solve the problem.
 - With the analytical solution, we may generate some insights.

²Technically, we should also have nonnegativity constraints $q_H \geq 0$ and $q_L \geq 0$. To make the presentation concise, however, I will hide these two constraints.

Step 1: Monotonicity

- ▶ By adding the two IC constraints

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L$$

and

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H,$$

we obtain

$$\begin{aligned}\theta_H v(q_H) + \theta_L v(q_L) &\geq \theta_H v(q_L) + \theta_L v(q_H) \\ \Rightarrow (\theta_H - \theta_L)v(q_H) &\geq (\theta_H - \theta_L)v(q_L) \\ \Rightarrow v(q_H) &\geq v(q_L) \\ \Rightarrow q_H &\geq q_L.\end{aligned}$$

- ▶ This is the **monotonicity** condition: In an incentive-compatible menu, the high-type consumer consume more.
 - ▶ Intuition: The high-type consumer prefers a high consumption.

Step 2: (IR-H) is redundant

- ▶ (IC-H) and (IR-L) imply that (IR-H) is redundant:

$$\begin{aligned}
 \theta_H v(q_H) - T_H &\geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\
 &> \theta_L v(q_L) - T_L && (\theta_H > \theta_L) \\
 &\geq 0. && \text{(IR-L)}
 \end{aligned}$$

- ▶ The high-type consumer earns **a positive rent**. Full surplus extraction is impossible under information asymmetry.
- ▶ The problem reduces to

$$\begin{aligned}
 \max_{q_H, T_H, q_L, T_L} & \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] && \text{(OBJ)} \\
 \text{s.t.} & \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L && \text{(IC-H)} \\
 & \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H && \text{(IC-L)} \\
 & \theta_L v(q_L) - T_L \geq 0. && \text{(IR-L)}
 \end{aligned}$$

Step 3: Ignore (IC-L)

- ▶ Let's “guess” that (IC-L) will be redundant and ignore it for a while.
 - ▶ Intuition: The low-type consumer **has no incentive** to pretend that he really likes the product.
 - ▶ We will verify that the optimal solution of the relaxed program indeed satisfies (IC-L).
- ▶ The problem reduces to

$$\max_{q_H, T_H, q_L, T_L} \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] \quad (\text{OBJ})$$

$$\text{s.t.} \quad \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \quad (\text{IC-H})$$

$$\theta_L v(q_L) - T_L \geq 0. \quad (\text{IR-L})$$

Step 4: Remaining constraints bind at optimality

$$\max_{q_H, T_H, q_L, T_L} \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] \quad (\text{OBJ})$$

$$\text{s.t. } \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \quad (\text{IC-H})$$

$$\theta_L v(q_L) - T_L \geq 0. \quad (\text{IR-L})$$

- ▶ (IC-H) must be **binding** at any optimal solution:
 - ▶ The seller wants to increase T_H as much as possible.
 - ▶ She will keep doing so until (IC-H) is binding.
- ▶ (IR-L) must also be **binding** at any optimal solution:
 - ▶ The seller wants to increase T_L as much as possible.
 - ▶ She will keep doing so until (IR-L) is binding.
 - ▶ Note that increasing T_L makes (IC-H) more relaxed rather than tighter.
- ▶ Note that if we did not ignore (IC-L), i.e.,

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H,$$

then we cannot claim that (IR-L) is binding!

Step 5: Removing the transfers

- ▶ The problem reduces to

$$\max_{q_H, T_H, q_L, T_L} \beta [T_L - cq_L] + (1 - \beta) [T_H - cq_H] \quad (\text{OBJ})$$

$$\text{s.t.} \quad \theta_H v(q_H) - T_H = \theta_H v(q_L) - T_L \quad (\text{IC-H})$$

$$\theta_L v(q_L) - T_L = 0. \quad (\text{IR-L})$$

- ▶ Therefore, we may remove the two constraints and replace T_L and T_H in (OBJ) by $\theta_L v(q_L)$ and $\theta_H v(q_H) - \theta_H v(q_L) + \theta_L v(q_L)$, respectively.
- ▶ The problem reduces to an **unconstrained** problem

$$\begin{aligned} \max_{q_H, q_L} & \quad \beta [\theta_L v(q_L) - cq_L] \\ & + (1 - \beta) [\theta_H v(q_H) - \theta_H v(q_L) + \theta_L v(q_L) - cq_H]. \end{aligned}$$

Step 6: Solving the unconstrained problem

- ▶ To solve

$$\max_{q_H, q_L} \beta \left[\theta_L v(q_L) - cq_L \right] + (1 - \beta) \left[\theta_H v(q_H) - cq_H - (\theta_H - \theta_L)v(q_L) \right],$$

note that because $v(\cdot)$ is strictly concave, the reduced objective function is strictly concave in q_H and q_L .

- ▶ If $\frac{\theta_H - \theta_L}{\theta_H} < \beta$, the **second-best** solution $\{(q_L^*, T_L^*), (q_H^*, T_H^*)\}$ satisfies the FOC:³

$$\theta_H v'(q_H^*) = c \quad \text{and} \quad \theta_L v'(q_L^*) = c \left[\frac{1}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{\theta_L} \right)} \right].$$

³If $\frac{\theta_H - \theta_L}{\theta_H} \geq \beta$, $q_L^* = 0$ and q_H^* still satisfies $\theta_H v'(q_H^*) = c$.

Step 7: Verifying that (IC-L) is satisfied

- ▶ To verify that (IC-L) is satisfied, we apply

$$T_L = \theta_L v(q_L) \quad \text{and} \quad T_H = \theta_H v(q_H) - (\theta_H - \theta_L)v(q_L).$$

- ▶ With this, (IC-L)

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H$$

is equivalent to

$$0 \geq -(\theta_H - \theta_L) \left[v(q_H) - v(q_L) \right].$$

With the monotonicity condition, (IC-L) is satisfied.

Inefficient consumption levels

- Recall that the first-best consumption levels \tilde{q}_L and \tilde{q}_H satisfy

$$\theta_H v'(\tilde{q}_H) = c \quad \text{and} \quad \theta_L v'(\tilde{q}_L) = c.$$

Moreover, the second-best consumption levels satisfy

$$\theta_H v'(q_H^*) = c \quad \text{and} \quad \theta_L v'(q_L^*) = c \left[\frac{1}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{\theta_L} \right)} \right] > c.$$

- The high-type consumer consumes the **first-best** amount.
- For the low-type consumer, $v'(\tilde{q}_L) = \frac{c}{\theta_L} < v'(q_L^*)$. As $v(\cdot)$ is strictly concave (so $v'(\cdot)$ is decreasing), $q_L^* < \tilde{q}_L$.
- The low-type consumer consumes **less** than the first-best amount.
 - Information asymmetry causes inefficiency.
 - The consumption will only decrease. It will not become larger. Why?

Cost of inducing truth-telling

- ▶ Regarding the consumption levels:
 - ▶ We have $q_L^* < \tilde{q}_L$. Why do we decrease q_L ?
 - ▶ Recall that under the first-best menu, the high-type consumer pretends to have a low valuation and earns $(\theta_H - \theta_L)v(\tilde{q}_L) > 0$.
 - ▶ Because he prefers a high consumption level, we must **cut down** q_L to make him **unwilling to lie**.
 - ▶ Inevitably, decreasing q_L creates inefficiency.
- ▶ Regarding the consumer surplus:
 - ▶ In equilibrium, the low-type consumer earns $\theta_L v(q_L^*) - T_L^* = 0$.
 - ▶ However, the high-type consumer earns

$$\theta_H v(q_H^*) - T_H^* = (\theta_H - \theta_L)v(q_L^*) > 0.$$

- ▶ The high-type consumer earns a positive **information rent**.
 - ▶ The agent earns a positive rent **in expectation**.
- ▶ Note that the high-type consumer's rent depends on q_L^* .
- ▶ Cutting down q_L^* is to cut down his information rent!

Summary

- ▶ We discussed a two-type monopoly pricing problem.
- ▶ We found the first-best and second-best mechanisms.
 - ▶ First-best: with complete information.
 - ▶ Second-best: under information asymmetry.
 - ▶ Thanks to the revelation principle!
- ▶ For the second-best solution:
 - ▶ **Monotonicity**: The high-type consumption level is higher.
 - ▶ **Efficiency at top**: The high-type consumption level is efficient.
 - ▶ **No rent at bottom**: The low-type consumer earns no rent.
- ▶ Information asymmetry **protects the agent**.
 - ▶ But it hurts the principal and social welfare.

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The idea of the revelation principle

- ▶ In general, the principal designs a mechanism for the agent(s).
 - ▶ The mechanism specifies a game rule. Agents act according to the rules.
- ▶ When agents have private types, there are two kinds of mechanisms.
- ▶ Under an **indirect mechanism**:
 - ▶ The principal specifies a function mapping agents' actions to payoffs.
 - ▶ Each agent, based on his type and his belief on other agents' types, acts to maximize his expected utilities.
- ▶ Under a **direct mechanism**:
 - ▶ The principal specifies a function mapping agents' **reported types** to actions and payoffs.
 - ▶ Each agent, based on his type and his belief on other agents' types, **reports a type** to maximize his expected utilities.
- ▶ If a direct mechanism can reveal agents' types (i.e., making all agents report truthfully), it is a **direct revelation mechanism**.

The idea of the revelation principle

Proposition 1 (Revelation principle)

Given any equilibrium of any given indirect mechanism, there is a direct revelation mechanism under which the equilibrium is equivalent to the given one: In the two equilibria, agents do the same actions.

- ▶ The idea is to “imitate” the given equilibrium.
- ▶ The given equilibrium specifies each agent’s (1) strategy to map his type to an action and (2) his expected payoff.
- ▶ We may “construct” a direct mechanism as follows:
 - ▶ Given any type report (some types may be false), find the **corresponding actions and payoffs** in the given equilibrium as if the agents’ types are really as reported.
 - ▶ Then assign **exactly those actions and payoffs** to agents.
- ▶ If the agents all report truthfully under the direct mechanism, they are receiving exactly what they receive in the given equilibrium. Therefore, under the direct mechanism no one deviates.