Information Economics, Fall 2016
Pre-lecture Problems for Lecture 9

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Note. The deadline of submitting the pre-lecture problem is ***9:20am, November 14, 2016***. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 points) Consider page 13 of the slides. Verify that the first-best prices are as stated.
2. (0 points) In both lemmas in the slides, the low-quality firm chooses its first-best prices. Intuitively explain why the low-quality firm's decision is not affected by the privatization of its quality level.
3. (10 points) A seller is going to sell a product of quality q at price p. The quality $q \in \{q_L, q_H\}$ is privately observed by the seller and is hidden to the consumers. The consumers believe that $\Pr(q = q_L) = \beta = 1 - \Pr(q = q_H)$ for some $\beta \in (0, 1)$. Consumers' willingness-to-pay θ is uniformly distributed between 0 and 1. A type-θ consumer buys the product if his utility $\theta\tilde{q} - p \geq 0$, where $\tilde{q}$ is the quality level in his belief. The unit production costs are c_H for the high-quality quality and c_L for the low-quality one. We normalize c_L to 0. It is publicly known that $q_H > q_L > 2c_H > 0$.

   (a) (2 points) Assuming that there is no information asymmetry, find the type-i seller's first-best price p_i^{FB}, $i \in \{L, H\}$.

   (b) (4 points) Assuming that there is information asymmetry, consider a separating equilibrium in which the high-quality seller's price p_H is different from that of the low-quality seller p_L. Suppose that the low-quality seller sets p_L to the first-best level. Convince yourself that the high-quality seller's optimization problem is

$$\begin{aligned}
\max_{p_H} \quad & \left(1 - \frac{p_H}{q_H}\right)(p_H - c_H) \\
\text{s.t.} \quad & \left(1 - \frac{p_L^{FB}}{q_L}\right)p_L^{FB} \geq \left(1 - \frac{p_H}{q_H}\right)p_H \\
& \left(1 - \frac{p_H}{q_H}\right)(p_H - c_H) \geq \left(1 - \frac{p_L^{FB}}{q_L}\right)(p_L^{FB} - c_H).
\end{aligned}$$

   Explain the meanings of the objective function and constraints in words.

   (c) (2 points) Show that the above program is feasible.

   Note. This implies a separating equilibrium exists.

   (d) (2 points) Show that the high-quality seller's first-best price p_H^{FB} always satisfies the second constraint.

   Note. This is intuitive, as the high-quality firm has no incentive to mimic the low-quality one.

   (e) (0 point) If you try to plug in p_H^{FB} into the first constraint, you will get the first constraint is satisfied if and only if $q_L q_H \geq q_H^2 - c_H^2$. When will this condition be satisfied? Intuitively explain why.

   Note. Note that when this condition is satisfied, there is no upward distortion in the price of the high-quality product!