Operations Research, Spring 2013
Homework 06

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1. (10 points) Consider the integer program

$$\begin{array}{rrcrcl} \max & 8x_1 & + & 5x_2 & & \\ \text{s.t.} & x_1 & + & x_2 & \leq & 6 \\ & 9x_1 & + & 5x_2 & \leq & 45 \\ & x_i \in \mathbb{Z}_+ & & \forall i = 1, 2. & & \end{array}$$

   we discussed in class. By branching on x_2 instead on x_1 at the first node, apply the branch-and-bound algorithm to solve this problem.

2. (Modified from Problem 9.3.3; 20 points) Use branch-and-bound to solve the following integer program:

$$\begin{array}{rl} \max & 2x_1 + 3x_2 \\ \text{s.t.} & x_1 + 2x_2 \leq 10 \\ & 3x_1 + 4x_2 \leq 25 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2 \end{array}$$

   The set $\mathbb{Z}$ is the set of all integers and the set $\mathbb{Z}_+$ is the set of all nonnegative integers. The notation "$x_i \in \mathbb{Z}_+$" therefore means "x_i is an integer and $x_i \geq 0$".

3. (Modified from Problem 9.5.2; 15 points) I am moving from New Jersey to Indiana and have rented a truck that can haul up to 1,100 cu ft of furniture. The volume and value of each item I am considering moving on the truck are given in the table below. Determine the items I should bring to Indiana by solving a knapsack problem. To solve this problem as a knapsack problem, what unrealistic assumptions must we make?

Item	Value ($)	Volume (cu ft)
Bedroom set	60	800
Dining room set	48	600
Stereo	14	300
Sofa	31	400
TV set	10	200

4. (Modified from Problem 9.2.12; 15 points) A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is $400 for New York, $500 for Los Angeles, $300 for Chicago, and $150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week, and region 3 requires 40 units per week. The costs (including production and shipping costs) of sending one unit from a plant to a region are shown in the table below.

From	To Region 1	To Region 2	To Region 3
New York	20	40	50
Los Angeles	48	15	26
Chicago	26	35	18
Atlanta	24	50	35

We want to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions:

- If the New York warehouse is opened, then the Los Angeles warehouse must be opened.
- At most two warehouses can be opened.
- Either the Atlanta or the Los Angeles warehouse must be opened.

Formulate an IP that can be used to minimize the weekly costs of meeting demand.

5. (Problem 9.Review.16; 20 points) Eastinghouse ships 12,000 capacitors per month to their customers. The capacitors may be produced at three different plants. The production capacity, fixed monthly cost of operation, and variable cost of producing a capacitor at each plant are given in the table below. The fixed cost for a plant is incurred only if the plant is used to make any capacitors.

Plant	Fixed cost ($)	Variable cost ($)	Capacity
1	80,000	20	6,000
2	40,000	25	7,000
3	30,000	30	6,000

   (a) (15 points) Formulate an integer programming model whose solution will tell Eastinghouse how to minimize their monthly costs of meeting their customers' demands.

   (b) (5 points) Use the MS Excel Solver to find the optimal plan. Include both your Solver formulation (by printing screen, for example) and the optimal plan as your answer.

6. (Problem 9.Review.21; 20 points) Gotham City has been divided into eight districts. The time (in minutes) it takes an ambulance to travel from one district to another is shown in the table below. The population of each district (in thousands) is as follows: district 1, 40; district 2, 30; district 3, 35; district 4, 20; district 5, 15; district 6, 50; district 7, 45; district 8, 60.

District	1	2	3	4	5	6	7	8
1	0	3	4	6	8	9	8	10
2	3	0	5	4	8	6	12	9
3	4	5	0	2	2	3	5	7
4	6	4	2	0	3	2	5	4
5	8	8	2	3	0	2	2	4
6	9	6	3	2	2	0	3	2
7	8	12	5	5	2	3	0	2
8	10	9	7	4	4	2	2	0

   (a) (5 points) Construct a 8×8 matrix A such that $A_{ij} = 1$ if the number of minutes for an ambulance to travel between districts i and j is no greater than 2 minutes.

   (b) (10 points) The city has only two ambulances and wants to locate them to maximize the number of people who live within 2 minutes of an ambulance. Utilize the matrix A you constructed to formulate an IP to achieve this goal.

   (c) (5 points) Use the MS Excel Solver to find the optimal plan. Include both your Solver formulation (by printing screen, for example) and the optimal plan as your answer.