Operations Research, Spring 2013
Homework 09

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. (Modified from Problem 15.2.1; 15 points, 3 points each) Each month, a gas station sells 4,000 gallons of gasoline. Each time the parent company refills the station's tanks, it charges the station a fixed cost \$50 plus a variable cost \$0.7 per gallon. The annual cost of holding a gallon of gasoline is \$0.3. Suppose the demand rate is constant and no shortage is allowed. Assume 1 week is $\frac{1}{52}$ year.

   (a) How large should the station's one order be?

   (b) How many orders per year will be placed in average?

   (c) How long will it be between orders?

   (d) If the lead time is two weeks, what is the reorder point?

   (e) If the lead time is ten weeks, what is the reorder point?

2. (10 points) Following from Problem 1, for quantity q from 1000 to 10000 gallons, depict the annual holding cost, annual ordering cost, and annual total cost (sum of the first two) as functions of q in one single figure. Show that the two curves of annual holding cost and annual ordering cost indeed intersect at the EOQ.

3. (Modified from Problem 15.2.8; 10 points) Consider an EOQ problem with annual demand D, unit holding cost h per year, and unit ordering cost K. Let $TC(q)$ be the total holding and ordering cost under an order quantity q and q^* be the optimal order quantity.

   (a) (2 points) Let $d \in (0, q^*)$ be a deviation from the optimal order quantity. What is $TC(q^* + d)$?

   (b) (2 points) Following Part (a), what is $TC(q^* - d)$?

   (c) (6 points) Prove that for any deviation $d \in (0, q^*)$, an order size of $q^* + d$ will have a lower annual total cost than an order size of $q^* - d$.

   Remark. The result in Part (c) provides the following managerial implication: It is more harmful to underestimate the optimal order quantity than to overestimate it. Therefore, if those parameters of the EOQ problem are just estimations, it is typically saver to order slightly *more* than the EOQ.

4. (Modified from Problem 15.2.9; 15 points) Consider an EOQ problem with annual demand D, unit holding cost h per year, and unit ordering cost K. Let $TC(q)$ be the total holding and ordering cost under an order quantity q and q^* be the optimal order quantity.

   (a) (10 points) Suppose that we order rq^* for some $r \in (0, 1)$. Show that

   $$f(r) = \frac{TC(rq^*)}{TC(q^*)} = \frac{1}{2}\left(\frac{1}{r} + r\right).$$

   (b) (5 points) Following Part (a), depict $f(r)$ for $r \in [\frac{1}{2}, 2]$. What are the values of $f(\frac{1}{2})$ and $f(2)$?

   Remark. The results here show that the EOQ model is *robust*, i.e., a huge over- or under-estimation in problem parameters only results in a relatively small deviation in the total cost. For example, suppose we overestimate the ordering cost by two times, i.e., we mistakenly think K is actually $2K$. In this case, we will order $\sqrt{2}q^*$, but the resulting total cost is just about 6% higher than that with no error in estimation. In short, using the EOQ model does not require a very precise estimation of parameters and those assumptions are allowed to be reasonably violated.

5. (Modified from Problem 15.4.2; 15 points, 5 points each) A company can produce 100 computers per day. The setup cost for a production run is \$1,000. The cost of holding a computer in inventory for one year is \$300. Customers demand 2,000 home computers per month (assume that one month is 30 days and there are 360 days in one year).

   (a) What is the optimal production lot size?

   (b) How many production runs must be made each year?

   (c) What is the optimal cycle time?

   (d) In a cycle, what is the proportion of time under production?

   (e) In a cycle, what is the proportion of time with no production?

6. (Modified from Problem 16.4.3; 10 points) Joe is selling Christmas trees to pay his college tuition. He purchases trees for \$10 each and sells them for \$25 each. The number of trees he can sell is normally distributed with a mean of 100 and standard deviation of 30. Use the newsvendor model to help Joe determine the number of trees he should purchase.

   Note. While you may feel that Joe should try to maximize his expected profit rather than minimizing his expected cost, in the last problem of this homework we show that these are equivalent, at least for a special case.

7. (Modified from Problem 16.4.4; 10 points, 5 points each) A hot dog vendor at Wrigley Field sells hot dogs for \$3 each. He buys them for \$1.20 each. All the hot dogs he fails to sell at Wrigley Field during the afternoon can be sold that evening at Comiskey Park for \$1 each. The daily demand for hot dogs at Wrigley Field is normally distributed with a mean of 40 and a standard deviation of 10.

   (a) If he buys 52 hot dogs, what is the probability that he will meet all of the day's demand for hot dogs at Wrigley?

   (b) If he buys hot dogs once a day, how many should he buy to minimize his expected cost?

8. (15 points) In this problem, we show that maximizing a newsvendor's expected profit is equivalent to minimizing its expected cost. To make our lives easier, let's consider only a special case in which the unit sales revenue is r, unit purchasing cost is c, and demand D is uniformly distributed between 0 and 1.

   (a) (0 point) Suppose the order quantity is q, the expected sales is $\mathbb{E}\big[\min\{D, q\}\big]$. With this, convince yourself that the profit-maximizing newsvendor's problem is

   $$\max_{q \geq 0} \; r\mathbb{E}\big[\min\{D, q\}\big] - cq.$$

   (b) (5 points) Rewrite $\mathbb{E}\big[\min\{D, q\}\big]$ as a function of q. Note that the pdf of D is $f(x) = 1$ for all $x \in [0, 1]$.

   (c) (5 points) Using the results in Parts (a) and (b), show that the profit-maximizing newsvendor's problem can be rewritten into

   $$\max_{q \geq 0} \; -\frac{r}{2}q^2 + (r - c)q.$$

   Then show that this is a convex program.

   (d) (5 points) Find the optimal order quantity q^* by solving the program in Part (c). Then show that it indeed satisfies $F(q^*) = \frac{c_u}{c_o + c_u}$, where c_o is the overage cost, c_u is the underage cost, and F is the cdf of $D \sim \text{Uni}(0, 1)$.