

# Operations Research, Spring 2013

## Project 2

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### The problem

A firm produces one product to satisfy known consumer demands in the next  $T$  periods. The demand is  $D_t$  and the unit production cost is  $C_t$  in period  $t$ ,  $t = 1, \dots, T$ . The unit holding cost per period is  $H$  (based on the ending inventory of each period). Moreover, if the firm wants to produce any positive amount of product in period  $t$ ,  $t = 1, \dots, 30$ , it must open the machines in its factory, and this requires the firm to pay a fixed setup cost  $S_t$  regardless of the production quantity. The demand in period  $t$  can be fulfilled by products produced in period  $t$  and the ending inventory of period  $t - 1$ , and the firm has  $I$  units of initial inventory at the beginning of period 1. For all problems below, please assume that the production quantities can be fractional.

1. Suppose all consumer demands must be fulfilled in time and the firm wants to minimize the total cost, including the production costs, holding costs, and setup costs in the  $T$  periods. Formulate a linear integer program for the firm. When you need “a large enough number”, you need to define it as a function of parameters, i.e.,  $D_t$ ,  $C_t$ , etc.
2. Following Problem 1, suppose now there is an additional cost “initialization cost”. If the production process is initiated in period  $t$ , i.e., the firm does not open the machines in period  $t - 1$  but starts to open them in period  $t$ , the firm must pay an initialization cost  $R_t$ ,  $t = 1, \dots, T$ .<sup>1</sup> Note that the firm may choose to keep machines open but produce nothing in a period to save future initialization costs. With the same objective, formulate a new linear integer program for the firm.
3. Following Problem 2, suppose now there is a new policy: Once the production process is initiated, it must last for at least three periods (i.e., two additional period).<sup>2</sup> With the same objective, formulate a new linear integer program for the firm.
4. Ignore what you have done in Problems 2 and 3. Following Problem 1, suppose now all demands must only be fulfilled by the end of period  $T$ . If a unit of demand of period  $t$  is not fulfilled in period  $t$ , a shortage cost  $U$  per period per unit is incurred.<sup>3</sup> Note that the firm can never fulfill demands in advance.
5. Following from Problem 4, use MS Excel Solver to find the optimal plan for the data given in “ORSp13\_project1.xlsx”.

### Submission and grading

Please form a group of no more than five people for this project. Submit a hard copy of your report to the instructor’s mail box at the first floor of the Management Building II by 1:00pm, May 9, 2013. The report cannot be longer than ten pages.

Your report will be graded based on the correctness (60%), organization and formatting (30%), creativity (10%), and the quality of oral presentation (bonus: 10%). All team members get the same grades. Among all teams that volunteer to present on May 13, 2013, at most five teams will be chosen based on the quality of the written report. The selection of teams for presentation will be announced shortly after the due date.

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<sup>1</sup>For example, if the factory decides to produce in periods 5, 6, 7, 12, 13, and 20, it not only pays  $S_5 + S_6 + S_7 + S_{12} + S_{13} + S_{20}$  as setup cost but also pays  $R_5 + R_{12} + R_{20}$  as the initialization cost.

<sup>2</sup>For example, it is not allowed to produce in periods 10, 11, 12, 15, and 16, because the production process initiated in period 15 does not last for at least three periods.

<sup>3</sup>For example, If  $T = 2$ ,  $D_1 = 10$ ,  $D_2 = 20$ ,  $D_3 = 30$ , and the production quantities of periods 1, 2, and 3 are 0, 5, 55, the total shortage cost to pay is  $(10 + 5 + 20)U$ : For the 10 units of demand in period 1, all of them are delayed for one period and then 5 of them are delayed for one more period. For the 20 units of demand in period 2, all of them are delayed for one period. You may see that using the 5 units produced in period 2 to fulfill either the demands in period 1 or 2 results in the same total shortage cost.