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Suggested Solution for Assignment 2

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

(a) Let the parameters be

$$
\begin{aligned}
c_{ij} &= \text{the bussing cost per student assigned from area } i \text{ to school } j,\ i = 1, ..., 6,\ j = 1, 2, 3, \\
a_j &= \text{the capacity in school } j,\ j = 1, 2, 3, \\
n_i &= \text{the total number of students in area } i,\ i = 1, ..., 6, \text{ and} \\
p_{ik} &= \text{the percentage in } k\text{th grade assigned from area } i,\ i = 1, ..., 6,\ k = 6, 7, 8.
\end{aligned}
$$

Let the decision variables be

$$x_{ij} = \text{the number of students assigned from area } i \text{ to school } j,\ i = 1, ..., 6,\ j = 1, 2, 3.$$

The objective is to minimize the total bussing cost. Therefore, we minimize

$$\sum_{i=1}^{6} \sum_{j=1}^{3} c_{ij} x_{ij}.$$

For each school, the total number of students assigned from six areas cannot exceed its capacity. Therefore, we have

$$\sum_{i=1}^{6} x_{ij} \leq a_j \quad \forall j = 1, 2, 3.$$

For each area, the total number of students assigned to three schools should be equal to the total number of students in the area. Therefore, we have

$$\sum_{j=1}^{3} x_{ij} = n_i \quad \forall i = 1, ..., 6.$$

For each school, the total number of students in each grade assigned from six area should not be less than 30 percent of the school's population. Therefore, we have

$$\sum_{i=1}^{6} p_{ik} x_{ij} \geq 0.3 \sum_{i=1}^{6} x_{ij} \quad \forall j = 1, 2, 3, k = 6, 7, 8.$$

For each school, the total number of students in each grade assigned from six area should not be greater than 36 percent of the school's population. Therefore, we have

$$\sum_{i=1}^{6} p_{ik} x_{ij} \leq 0.36 \sum_{i=1}^{6} x_{ij} \quad \forall j = 1, 2, 3, k = 6, 7, 8.$$

Therefore, the complete formulation of the problem is

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{6}\sum_{j=1}^{3} c_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{6} x_{ij} \le a_j \quad \forall j = 1,2,3 \\
& \sum_{j=1}^{3} x_{ij} = n_i \quad \forall i = 1,...,6 \\
& \sum_{i=1}^{6} p_{ik}x_{ij} \ge 0.3\sum_{i=1}^{6} x_{ij} \quad \forall j = 1,2,3, k = 6,7,8 \\
& \sum_{i=1}^{6} p_{ik}x_{ij} \le 0.36\sum_{i=1}^{6} x_{ij} \quad \forall j = 1,2,3, k = 6,7,8 \\
& x_{ij} \ge 0 \quad \forall i = 1,...,6, j = 1,2,3 \\
& x_{21} = 0, x_{52} = 0, x_{43} = 0.
\end{aligned}
$$

(b) By Microsoft Excel Solver, we get the optimal objective value (the minimal bussing cost) \$555555.8869, and the optimal solution (the optimal assignments) is shown in below:

Number of students

Area	School 1	School 2	School 3	Total
1	0	450	0	450
2	0	422.22	177.78	600
3	0	227.78	322.22	550
4	350	0	0	350
5	366.67	0	133.33	500
6	83.33	0	366.67	450
Total	800	1100	1000	

(c) Since the optimal solution we get in (b) contains decimals, we manually adjust each assignment to integer. The school board should assign students to schools in the way as shown in below for each grades, and the bussing cost would be \$555300.

Number of students in 6th

Area	School 1	School 2	School 3	Total
1	0	144	0	144
2	0	156	66	222
3	0	68	97	165
4	98	0	0	98
5	143	0	52	195
6	28	0	125	153
Total	269	368	340	

Number of students in 7th

Area	School 1	School 2	School 3	Total
1	0	171	0	171
2	0	118	50	168
3	0	68	97	176
4	140	0	0	140
5	125	0	45	170
6	23	0	103	126
Total	288	362	301	

Number of students in 8th

Area	School 1	School 2	School 3	Total
1	0	135	0	135
2	0	148	62	210
3	0	87	122	209
4	112	0	0	112
5	99	0	36	135
6	32	0	139	171
Total	243	370	359	

(d) Let the parameters be

c_{ij} = the bussing cost per student assigned from area i to school j, $i = 1, ..., 6, j = 1, 2, 3,$
a_j = the capacity in school j, $j = 1, 2, 3$, and
n_i = the total number of students in area i, $i = 1, ..., 6,$

Let the decision variables be

x_{ij} = the number of students assigned from area i to school j, $i = 1, ..., 6, j = 1, 2, 3,$

$$y_{ij} = \begin{cases} 1 & \text{if there is any student assigned from area } i \text{ to school } j \\ 0 & \text{otherwise} \end{cases} \quad , i = 1, ..., 6, j = 1, 2, 3.$$

The complete formulation of the problem is

$$\begin{aligned}
\min \quad & \sum_{i=1}^{6} \sum_{j=1}^{3} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{6} x_{ij} \leq a_j \quad \forall j = 1, 2, 3 \\
& \sum_{j=1}^{3} x_{ij} = n_i \quad \forall i = 1, ..., 6 \\
& x_{ij} = n_i y_{ij} \quad \forall i = 1, ..., 6, \ j = 1, 2, 3 \\
& \sum_{j=1}^{3} y_{ij} = 1 \quad \forall i = 1, ..., 6 \\
& x_{ij} \geq 0 \quad \forall i = 1, ..., 6, \ j = 1, 2, 3 \\
& x_{21} = 0, x_{52} = 0, x_{43} = 0.
\end{aligned}$$

To ensure that all the students in each area is assigned to just one school, we have constraints $\sum_{j=1}^{3} y_{ij} = 1$ and $x_{ij} = n_i y_{ij}$. The first constraint $\sum_{j=1}^{3} y_{ij} = 1$ guarantees only one school be assigned to, and the second constraint $x_{ij} = n_i y_{ij}$ guarantees all the students in the area be assigned together. By Microsoft Excel Solver, the school board should assign students to schools in the way as shown in below (the optimal solution), and the optimal objective value (the bussing cost) would be \$420000, the total bussing cost decreases \$135300.

Number of students

Area	School 1	School 2	School 3	Total
1	0	450	0	450
2	0	600	0	600
3	0	0	550	550
4	350	0	0	350
5	500	0	0	500
6	0	0	450	450
Total	850	1050	1000	

(e) The linear programming model is the same as the one in part (a) except for modifying bussing cost c_{41} and c_{33} to 0. By Microsoft Excel Solver, we get the optimal objective value (the minimal bussing cost) \$393636.3636, and the optimal solution (the optimal assignments) is shown in below:

Number of students

Area	School 1	School 2	School 3	Total
1	0	450	0	450
2	0	600	0	600
3	0	0	550	550
4	350	0	0	350
5	318.18	0	181.82	500
6	131.82	50	268.18	450
Total	800	1100	1000	

(f) The linear programming model is the same as the one in part (a) except for modifying bussing cost $c_{11}, c_{41}, c_{32}, c_{62}$, and c_{33} to 0. By Microsoft Excel Solver, we get the optimal objective value (the minimal bussing cost) \$340053.7634, and the optimal solution (the optimal assignments) is shown in below:

Number of students

Area	School 1	School 2	School 3	Total
1	38.71	411.29	0	450
2	0	236.56	363.44	600
3	0	77.96	472.04	550
4	350	0	0	350
5	435.48	0	64.52	500
6	75.81	374.19	0	450
Total	900	1100	900	