Operations Research, Spring 2014
Homework 1

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Note. The deadline of this homework is ***1pm, February 27, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (0 points) Write down the names of your teammates for the following two weeks.
2. (40 points; 10 points each) Nowadays The Institute for Operations Research and the Management Sciences (INFORMS) is the major academic society of operations researchers and management scientists. Please answer the following questions with your MOTHER LANGUAGE by first reading https://www.informs.org/About-INFORMS/What-is-Operations-Research. Please note that the information there may not be enough for making good answers.
   (a) What is the relationship between operations research and mathematical models? Explain in your own words and limit your answer to 200 words.
   (b) What is the relationship between operations research and management science? Explain in your own words and limit your answer to 200 words.
   (c) Among those sub-societies in INFORMS, two are marketing science and revenue management. Describe one marketing decision that may be supported by operations research in your own word. Limit your answer to 500 words.
   (d) Among those theories mentioned on this page, one is game theory. Describe one decision-making environment in which other decision makers' decisions greatly affect the outcome of your own decision. Explain in your own words and limit your answer to 500 words.
3. (20 points; 5 points each) Solve the following problems regarding linear algebra.
   (a) Consider a linear system
   $$\begin{array}{rcrcrcl} & & 2x_2 & + & 2x_3 & = & 4 \\ x_1 & + & 2x_2 & + & x_3 & = & 4 \\ & & x_2 & - & x_3 & = & 0. \end{array}$$
   Use Gauss-Jordan elimination to determine whether the system has a unique solution, infinitely many solutions, or no solution. In the first two cases, write down one solution.
   (b) Use Gauss-Jordan elimination to find the inverse (or show there is no inverse) of the matrix
   $$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}.$$
   (c) Are the column vectors of the matrix in Part (b) linearly independent? Why or why not?
   (d) For the matrix
   $$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & -1 & 2 & 3 \\ 4 & 5 & 2 & 5 \end{bmatrix},$$
   what is the rank?
4. (20 points; 5 points each) Solve the following problems regarding probability. You must write down essential derivations that supports your answers.
   (a) Let X be the number you get by rolling a fair dice. What is $\mathbb{E}[X]$, the expectation of X?

(b) Continue from Part (a), what is $\mathrm{Var}(X)$, the variance of X?

(c) Let X_1 and X_2 be the numbers you get by rolling two fair dices. What is $\mathbb{E}[X_1 + X_2]$?

(d) Continue from Part (c), what is $\Pr(X_1 = X_2)$, the probability for X_1 and X_2 to be identical?

5. (20 points; 10 points each) Consider the problem of selling used textbooks discussed in class.

(a) Suppose one additional restriction is that either *Operations Research* or *Computer Programming* must be kept (i.e., you may not sell both of them). Formulate this restriction as a constraint with the decision variables defined in the slides. Then solve the new model and make suggestions.

(b) Continue from Part (a). Suppose another textbook *Optimization* weighs 2kg and may be sold at \$200. Follow the DFSI principle to solve this new problem.