Operations Research, Spring 2014
Homework 3

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Note. The deadline of this homework is ***1pm, March 13, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (10 points) Convert the following LP to its standard form:

$$\begin{aligned} \min \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 \geq 3 \\ & x_1 + x_2 \geq -4 \\ & 2x_1 - x_2 = 3 \\ & x_1 \leq 0, x_2 \text{ urs.} \end{aligned}$$

2. (15 points) Consider the following LP

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \geq 100 \\ & x_1 + x_2 \leq 80 \\ & x_1 \geq 40 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   (a) (10 points) Find all the bfs.

   (b) (5 points) Show how each bfs corresponds to an extreme point.

3. (15 points) Consider the following LP

$$\begin{aligned} z^* = \min \quad & 2x_1 - x_2 - x_3 \\ \text{s.t.} \quad & 3x_1 + x_2 + x_3 \leq 60 \\ & x_1 - x_2 + 2x_3 \leq 10 \\ & x_1 + x_2 - x_3 \leq 20 \\ & x_i \geq 0 \quad \forall\, i = 1, ..., 3. \end{aligned}$$

   Use the simplex method to find an optimal solution $x^* = (x_1^*, x_2^*, x_3^*)$ and the associated objective value z^*. Write down the complete process.

4. (20 points; 10 points each) Consider the following LP

$$\begin{aligned} z^* = \max \quad & 3x_1 + 2x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 100 \\ & x_1 + x_2 \leq 80 \\ & x_1 \leq 40 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   (a) Use the simplex method to find an optimal solution $x^* = (x_1^*, x_2^*)$ and the associated objective value z^*.

   (b) Depict the route you go through in Part (a).

5. (10 points; 5 points each) Explain the following concepts.

   (a) For a maximization LP, why do we choose a variable with a negative reduced cost to enter?

   (b) Why do we choose a variable with the minimum ratio to leave?

6. (30 points; 10 points each) A company produces A, B, and C and can sell these products in unlimited quantities at the following unit prices: A, \$9; B, \$50; C, \$100. Producing a unit of A requires 1 hour of labor. Producing a unit of B requires 2 hours of labor plus 2 units of A. Producing a unit of C requires 3 hours of labor plus 1 unit of B. Any A that is used to produce B cannot be sold. Similarly, any B that is used to produce C cannot be sold. A total of 40 hours of labor are available.

   (a) Let

   $$x_i = \text{sales quantity of product } i, i \in \{\text{A}, \text{B}, \text{C}\}.$$

   be the decision variables. Formulate an LP that can maximize the company's revenues.

   (b) Let

   $$y_i = \text{production quantity of product } i, i \in \{\text{A}, \text{B}, \text{C}\}.$$

   be the decision variables. Formulate an LP that can maximize the company's revenues.

   (c) Show that the above two models are equivalent (i.e., their optimal solutions will result in the same production and sales plan).