Operations Research, Spring 2014
Homework 10

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Note. The deadline of this homework is ***1pm, June 5, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (10 points; 5 points each) Consider the two-part tariff we introduced in class. The manufacturer's problem is

$$\begin{aligned} \pi_{\mathrm{M}}^{**} = \max_{w\geq 0, t\geq 0} \quad & (w-C)\left(\frac{A-Bw}{2}\right)+t \\ \text{s.t.} \quad & \frac{(A-Bw)^2}{4B}-t\geq 0. \end{aligned}$$

   It has been explained that the constraint must be binding at any optimal solution.[1]

   (a) Replace t by $\frac{(A-Bw)^2}{4B}$ in the objective function to reduce the problem to a single-variate problem. Show that the reduced problem is a convex program.

   (b) Apply the FOC to solve the problem.

2. (30 points) Consider a distribution channel in which a manufacturer (she) sells a product to a retailer (he), who then sells to end consumers. The manufacturer offers a contract and then the retailer decides an inventory level. Suppose that the unit production cost is \$10, the unit retail price is \$50, and the random demand follows a uniform distribution between 0 and 100.

   (a) (5 points) Find the efficient inventory level and the efficient expected channel profit $\pi_{\mathrm{C}}^{\mathrm{FB}}$.

   (b) (5 points) Under a wholesale contract, find the equilibrium outcome (including the wholesale price, inventory level, manufacturer's expected profit $\pi_{\mathrm{M}}^{(0)}$, retailer's expected profit $\pi_{\mathrm{R}}^{(0)}$, and expected channel profit $\pi_{\mathrm{C}}^{(0)}$). Prove or disprove that $\pi_{\mathrm{C}}^{(0)} < \pi_{\mathrm{C}}^{\mathrm{FB}}$.

   (c) (10 points) Under a return contract with wholesale price \$30 and return credit \$5, find the equilibrium inventory level, manufacturer's expected profit $\pi_{\mathrm{M}}^{(1)}$, retailer's expected profit $\pi_{\mathrm{R}}^{(1)}$, and expected channel profit $\pi_{\mathrm{C}}^{(1)}$. Is win-win achieved with this return contract (i.e., $\pi_{\mathrm{M}}^{(1)} > \pi_{\mathrm{M}}^{(0)}$ and $\pi_{\mathrm{R}}^{(1)} > \pi_{\mathrm{R}}^{(0)}$)?

   (d) (10 points) Under a return contract with wholesale price \$30 and return credit \$10, find the equilibrium inventory level, manufacturer's expected profit $\pi_{\mathrm{M}}^{(2)}$, retailer's expected profit $\pi_{\mathrm{R}}^{(2)}$, and expected channel profit $\pi_{\mathrm{C}}^{(2)}$. Is win-win achieved with this return contract (i.e., $\pi_{\mathrm{M}}^{(2)} > \pi_{\mathrm{M}}^{(1)}$ and $\pi_{\mathrm{R}}^{(2)} > \pi_{\mathrm{R}}^{(1)}$)?

3. (20 points) In a revenue sharing contract (w, ϕ) in a manufacturer-retailer relationship, the manufacturer charges the retailer $w \geq 0$ per unit of product and the retailer gives ϕs to the manufacturer if the sales revenue is s for some $\phi \in [0, 1]$. Suppose the retailer is a newsvendor facing a random demand $D \sim F, f$, the unit production cost is c, and the unit retail price is p.

   (a) (5 points) How is the wholesale contract a special case of the revenue sharing contract?

   (b) (10 points) Formulate the retailer's ordering problem given w and ϕ. Then find the retailer's optimal order quantity $q^*(w, \phi)$ as a function of w and ϕ.

[1] The constraint was made wrong as $\frac{(A-Bw)^2}{4} - t \geq 0$ during the lecture. It has been corrected on the slides now.

(c) (5 points) According to Part (b), find what kind of revenue sharing contracts coordinate the channel or show that there is none.

4. (25 points) Consider the chain-to-chain competition discussed in class with the ID industry equilibrium. Mathematically verify the two manufacturers' equilibrium expected profits shown on the slides.

5. (15 points) Consider a chain-to-chain competition model with two manufacturers setting their wholesale prices simultaneously and then two retailers setting their order quantities, not retail prices, simultaneously. In other words, the competition at the retail side is a Cournot competition rather than a Bertrand one. With no mathematical derivation, intuitively explain whether it is possible for DD to be an equilibrium industry structure.