

# Operations Research, Spring 2015

## Suggested Solution for Case Study 1

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1. (a) The model is given in solution of hw1. Check the AMPL model file "case1-1.mod" and data file "case1-1.dat". After solving, the maximum profit is 5464000 and we get

variables	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
$s_i$	2500	4000	4500	3600	3600	3600
$h_i$	200	2000	2000	2000	2000	2000
$t_i$	1600	1600	1600	1600	1600	1600
$x_i$	1300	900	0	0	0	0

2. (a) Let the parameters be

$I$  = the number of fabs, that is 8,  
 $J$  = the number of items, that is 12,  
 $K$  = the number of orders, that is 10,  
 $R_k$  = Revenue from order  $k$  if it's completed,  $k = 1, \dots, K$ ,  
 $H_{ij}$  = the production rate that Fab  $i$  produces item  $j$ ,  $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ,  
 $Q_{jk}$  = the demand quantity of item  $j$  in Order  $k$ ,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ ,  
 $C_i$  = the capacity of Fab  $i$ ,  $i = 1, \dots, I$ .

Let the decision variables be

$x_{ijk}$  = the production hour that Fab  $i$  produces item  $j$  in order  $k$ ,  $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ .

$$\begin{aligned}
 \max \quad & \sum_{k=1}^K R_k \left( \frac{\sum_{j=1}^J \sum_{i=1}^I x_{ijk} H_{ij}}{\sum_{j=1}^J Q_{jk}} \right) \\
 \text{s.t.} \quad & \sum_{k=1}^K \sum_{j=1}^J x_{ijk} \leq C_i \quad \forall i = 1, \dots, I \\
 & \sum_{i=1}^I x_{ijk} H_{ij} \leq Q_{jk} \quad \forall k = 1, \dots, K, j = 1, \dots, J \\
 & x_{ijk} \geq 0 \quad \forall i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K.
 \end{aligned}$$

- (b) Check the AMPL model file "case1-2.mod" and data file "case1-2.dat". After solving, the maximum profit is 70900 and we get

**Fab1**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	195	80	150	0	0	0
4	0	0	0	0	0	0	0	0	200	0	0	0
5	0	0	0	0	0	0	0	0	62.5	0	0	0
6	0	0	0	0	0	0	0	60	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	40	12.5	0	0	0
10	0	0	0	0	0	0	0	40	0	0	0	0
											sum	<u>840</u>

**Fab2**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	200	0	0
2	0	0	0	0	0	0	0	0	0	0	150	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	80	0	0
9	0	0	0	0	0	0	0	0	0	120	0	0
10	0	0	0	0	0	0	0	0	0	120	0	0
											sum	<u>670</u>

**Fab3**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	57.5	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	50	116.67	0	0	0	0	0	0	0	0	0
7	0	0	16.67	0	0	0	0	0	0	0	0	0
8	0	100	0	0	0	0	0	0	0	0	0	0
9	0	0	100	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	133.33
											sum	<u>574.167</u>

**Fab4**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	40	0	0	0	0	25	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	192.5	0	0	0	0	62.5	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	125	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	25	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
											sum	<u>470</u>

**Fab5**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	133.33	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	36.67	0	0	0	0	0
4	0	190	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	66.67	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	33.33	0	0	0	0	0
9	0	0	0	0	0	0	100	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
											sum	<u>560</u>

**Fab6**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	37.5	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	4.6875	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	50	0	0	0	20	0	0	0	0	0	0	0
9	37.5	0	0	0	10	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
											sum	<u>159.6875</u>

**Fab7**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	200	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	333.33	0	0	0	0	0	133.33	0
5	0	0	0	0	100	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	100	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	200	0
9	0	0	0	0	0	0	0	0	0	0	33.33	0
10	0	0	0	0	0	0	0	0	0	0	0	0
											sum	<u>1100</u>

**Fab8**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	66.67	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	100
4	0	0	0	0	0	0	0	0	0	0	0	150
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	50	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	25
8	0	0	0	16.67	0	0	0	0	0	0	0	50
9	0	0	0	133.33	0	0	0	0	0	0	0	75
10	0	0	0	66.67	0	0	0	0	0	0	0	0
											sum	<u>733.33</u>

3. (a) Let the parameters be

$I$  = the number of fabs, that is 8,

$J$  = the number of items, that is 12,

$K$  = the number of orders, that is 10,

$H_{ij}$  = the production rate that Fab  $i$  produces item  $j$ ,  $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ,

$Q_{jk}$  = the demand quantity of item  $j$  in Order  $k$ ,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ ,

$C_i$  = the capacity of Fab  $i$ ,  $i = 1, \dots, I$ .

Let the decision variables be

$x_{ijk}$  = the production hour that Fab  $i$  produces item  $j$  in order  $k$ ,  $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ .

$$\min \sum_{j=1}^J \sum_{i=1}^I x_{ij1} + \sum_{j=1}^J \sum_{i=1}^I x_{ij2} + \sum_{j=1}^J \sum_{i=1}^I x_{ij3} + \sum_{j=1}^J \sum_{i=1}^I x_{ij4} + \sum_{j=1}^J \sum_{i=1}^I x_{ij6} + \sum_{j=1}^J \sum_{i=1}^I x_{ij10}$$

$$\text{s.t.} \quad \sum_{k=1}^K \sum_{j=1}^J x_{ijk} \leq C_i \quad \forall i = 1, \dots, I$$

$$x_{1j1}H_{1j} + x_{2j1}H_{2j} \geq Q_{j1} \quad \forall j = 1, \dots, J$$

$$x_{2j2}H_{2j} + x_{3j2}H_{3j} \geq Q_{j2} \quad \forall j = 1, \dots, J$$

$$x_{3j3}H_{3j} + x_{4j3}H_{4j} + x_{5j3}H_{5j} + x_{7j3}H_{7j} \geq Q_{j3} \quad \forall j = 1, \dots, J$$

$$x_{4j4}H_{4j} + x_{5j4}H_{5j} + x_{7j4}H_{7j} + x_{8j4}H_{8j} \geq Q_{j4} \quad \forall j = 1, \dots, J$$

$$x_{5j6}H_{5j} + x_{6j6}H_{6j} + x_{7j6}H_{7j} \geq Q_{j6} \quad \forall j = 1, \dots, J$$

$$x_{1j10}H_{1j} + x_{3j10}H_{3j} + x_{5j10}H_{5j} + x_{6j10}H_{6j} + x_{7j10}H_{7j} + x_{8j10}H_{8j} \geq Q_{j10} \quad \forall j = 1, \dots, J$$

$$x_{ijk} \geq 0 \quad \forall i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K.$$

(b) Check the AMPL model file "case1-3.mod" and data file "case1-3.dat". After solving, the minimum machine hours is 4639.167 and we get

**Fab1**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	200	600	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	40	0	0	0	0
sum												840

**Fab2**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	200	0	0
2	200	0	0	0	0	200	0	0	0	0	150	0
sum												750

**Fab3**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
2	0	100	0	0	0	0	0	0	0	0	0	0
3	0	0	50	0	0	0	0	193.33	200	0	0	66.67
sum												610

**Fab4**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
4	0	0	0	0	200	0	0	0	0	0	66.67	0
											sum	266.67

**Fab5**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
3	0	0	0	0	0	0	166.67	0	0	0	0	0
4	0	238.33	0	0	0	0	0	0	0	0	0	0
6	0	66.67	88.33	0	0	0	0	0	0	0	0	0
											sum	560

**Fab6**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
6	0	0	43.33	30	0	0	0	100	0	0	0	0
10	0	0	0	0	0	0	0	0	0	66.67	0	0
											sum	240

**Fab7**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
3	0	0	0	0	0	0	0	413.33	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	100	0
											sum	513.33

**Fab8**

order/item	1	2	3	4	5	6	7	8	9	10	11	12
4	0	42.5	0	0	0	0	0	0	400	0	0	150
10	0	0	0	66.67	0	0	0	0	0	0	0	200
											sum	859.167

4. omitted