

# Operations Research

## Overview

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# Welcome!

- ▶ This is an **introductory** Operations Research course designed for second-year students majoring in **Information Management**.
- ▶ My plan for today:
  - ▶ Ch. 1: What is Operations Research?
  - ▶ Syllabus and quiz.
  - ▶ An in-class planning game.
  - ▶ Ch. 2: Introduction to modeling.
  - ▶ Feedback from 2015.



(by Hsuan Jung Chou)

# What is Operations Research?

- ▶ **Operations Research** (OR) is:
  - ▶ the methodology to “**allocate** the available **resources** to the various activities in a way that is most effective for the organization as a whole.”
  - ▶ “applied to problems that concern how to conduct and coordinate the **operations** (i.e., activities) within an organization.”<sup>1</sup>
- ▶ It aims to **support decision making**.
  - ▶ Typical tools: intuitions, business senses, and experiences.
  - ▶ And OR (and other quantitative tools)!
  - ▶ By doing OR studies, we generate some suggestions to **decision makers**.

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<sup>1</sup>Both quoted from *Introduction to Operations Research* by Hillier and Lieberman, the ninth edition.

## Industry applications



- ▶ Important questions:
  - ▶ How to deliver 6.5 millions items to more than 220 countries each day?
  - ▶ In each region, where to build distribution hubs?
  - ▶ In each distribution hub, how to classify and sort items?
  - ▶ In each city, how to choose routes?
- ▶ What do you need?
  - ▶ Well-designed information systems.
  - ▶ Operations Research!
- ▶ Further reading:
  - ▶ The application vignette in Section 1.4.
  - ▶ The article on CEIBA with the complete story.

# Industry applications



- ▶ Important questions:
  - ▶ How to determine the cities to connect?
  - ▶ How to schedule more than 2000 flights per day?
  - ▶ How to assign crews to flights?
  - ▶ How to reassign crews immediately when there is an emergency?
- ▶ What do you need?
  - ▶ Well-designed information systems.
  - ▶ Operations Research!
- ▶ Further reading:
  - ▶ The application vignette in Section 2.2.
  - ▶ The article on CEIBA with the complete story.

## Applicability and limitations

- ▶ It aims to support **decision making** in a **complicated** environment.
  - ▶ It is useless if we do not make decisions.
  - ▶ It is helpful if intuitions and experiences are not enough.
  - ▶ It is required if one's organization has many operations involved.
- ▶ It is a collection of **mathematical** (quantitative) methods.
  - ▶ Many methods come from **economics** and **computer science**.
  - ▶ It overlaps a lot with Management Science and Industrial Engineering.
- ▶ It is best for **quantifiable decisions**.
  - ▶ Those things that can be counted or measured.
  - ▶ E.g., quantities to produce, inventory to stock, amount to invest, routes to go, workers to assign, etc.
  - ▶ It is not so helpful for qualitative decisions.
- ▶ It almost always requires **computers**.
  - ▶ So that large-scale computations are possible.

## Summary

- ▶ What is Operations Research?
- ▶ We use **engineering** approaches to solve **managerial** problems.
  - ▶ A field of applied mathematics for making **better business decisions**.
- ▶ Operations Research is one of the few courses that lie in the **interface** between Business and Computer Science.
- ▶ It is a promising direction if you:
  - ▶ Want to learn something that help you do business operations **without** requiring a lot of experiences and domain knowledge.
  - ▶ Will work on **mathematical problems** in Computer Science, Economics, Operations Management, Finance, and many other fields.

## Before we start...

- ▶ If you are an IM student:
  - ▶ Do not take this course in your first year.
  - ▶ I will keep teaching this course before you graduate.
- ▶ If you are not:
  - ▶ Always welcome but think twice (or three times)!
  - ▶ After this three-hour lecture, complete an online form to ask for a registration code.



# Agenda

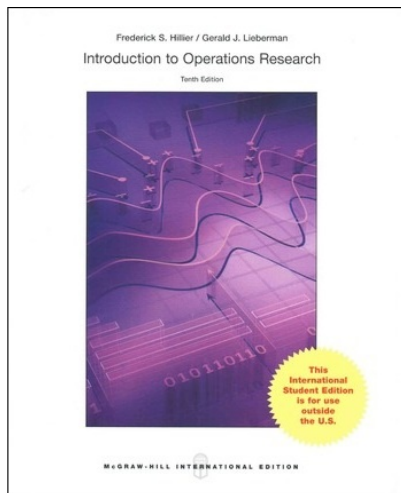
- ▶ Ch. 1: What is Operations Research?
- ▶ **Syllabus and quiz.**
- ▶ An in-class planning game.
- ▶ Ch. 2: Introduction to modeling.
- ▶ Feedback from 2015.

# Schedule

Week	Lecture	Deadline
1	Overview and quiz	
2	Linear Programming (LP)	CA 1: 2/29 (Mon)
3	The Simplex Method: Basics	
4	The Simplex Method: Advances	
5	Linearization Techniques & Optimization Software	HW 1: 3/21 (Mon)
6	LP Duality	CA 2: 3/28 (Mon)
7	Integer Programming (IP)	
8	Network Flow Models	
9	Applications of IP	
10	<b><i>In-class Problem-solving Challenge</i></b>	HW 2: 4/25 (Mon)
11	Single-variate Nonlinear Programming (NLP)	CA 3: 5/2 (Mon)
12	Multi-variate NLP: Analysis	
13	Multi-variate NLP: Algorithms	
14	No Class (Self-study on Self-selected topics)	HW 3: 5/23 (Mon)
15	<b><i>Final exam</i></b>	
16	No Class (Dragon Boat Festival)	
17	<b><i>Final Project Presentations</i></b>	Report: 6/17 (Fri)
18	No Class (Thanks to “Flipped Classroom”)	

# Textbook

- ▶ *Introduction to Operations Research* by Hillier and Lieberman, tenth edition, McGraw Hill.
- ▶ Reading the textbook will really let you learn a lot.
- ▶ Older versions are also helpful.



## “Flipped classroom” and self learning

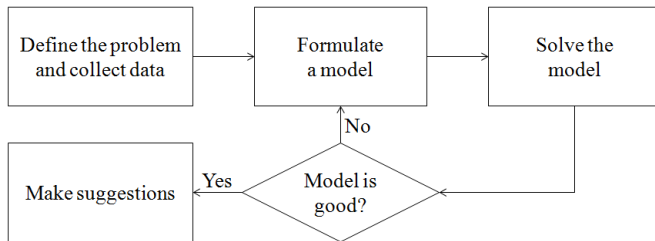
- ▶ “Flipped classroom”: “Lectures at home, class activities in class.”
  - ▶ Spend around 1–1.5 hours to watch lecture videos before a lecture.
  - ▶ Ask questions, do exercises, discuss with classmates and the instructor, and get advanced challenges in class.
  - ▶ No weekly homework anymore. Enjoy additional holidays.
  - ▶ Will actually save your time.
- ▶ Self learning:
  - ▶ One lecture will be canceled for self study.
  - ▶ A few advanced topics will be offered for students to teach themselves.
  - ▶ Tentative topics: Approximation algorithms, nonlinear optimization for logistic regression, Lagrange relaxation for distributed optimization.

# Agenda

- ▶ Ch. 1: What is Operations Research?
- ▶ Syllabus and quiz.
- ▶ An in-class planning game.
- ▶ **Ch. 2: Introduction to modeling.**
- ▶ Feedback from 2015.

## Five steps of an OR study

- ▶ To apply OR to facilitate better decision making, we conduct **OR studies** in five steps:
  - ▶ Define the problem and collect relevant data.
  - ▶ Formulate a **mathematical model** to represent the problem.
  - ▶ Develop or apply a procedure to derive a solution from the model.
  - ▶ Test the model and refine it when needed.
  - ▶ Make managerial suggestions.



- ▶ One thing should be defined: What is a mathematical model?

# Mathematical modeling

- ▶ The main “weapon” we will use in OR is **mathematical modeling**.
  - ▶ Often a mathematical model is called a **model**, a **formulation**, or a **program** in OR.
- ▶ Modeling is a way of **abstracting** a physical problem into a model with **symbols** and **formulas**.
  - ▶ Use mathematics to describe a problem.
- ▶ Why modeling?
  - ▶ We use a model to describe a problem **precisely** and **concisely**.
  - ▶ Once an **algorithm** for a type of model is developed, all problems that can be modeled in that way can be solved.

## An example: step 1

- ▶ Consider the following example.
  - ▶ I have three used textbooks to sell in a second-hand market.
  - ▶ I need to bring them to the market.
  - ▶ But I may carry at most 5 kg.
  - ▶ Which book(s) should I bring?
- ▶ Step 1: Define the problem and collect relevant data.
  - ▶ The problem: To maximize the sales revenue without hurting me.
  - ▶ Data:

Book	Title	Price (NT\$)	Weight (kg)
1	Calculus	500	4
2	Computer Programming	400	2
3	Operations Research	200	3



## Step 2: formulating the problem

- ▶ Step 2: Precisely **formulate** (i.e., describe) the problem.
- ▶ To describe a problem:
  - ▶ **Parameters**: What cannot be controlled by us?
  - ▶ **Decision variables**: What may we control?
  - ▶ **Objective function**: What do we want?
  - ▶ **Constraints**: What are the limitations?
- ▶ Parameters:
  - ▶ 5 kg and 3 books; 500, 400, and 200 dollars; 4 kg, 2 kg, and 3 kg.
- ▶ Decision variables:
  - ▶ For each book, we may control whether to bring it. We thus define

$$x_i = \begin{cases} 1 & \text{if I carry book } i \\ 0 & \text{otherwise} \end{cases}, i = 1, \dots, 3$$

as our decision variables.

## Step 2: formulating the problem

- ▶ What do we want? We want to maximize the sales revenue:

$$500x_1 + 400x_2 + 200x_3.$$

- ▶ What prevent us from bringing everything? We are not strong enough:

$$4x_1 + 2x_2 + 3x_3 \leq 5.$$

- ▶ Our first model:

$$\begin{array}{llllllll} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5. \end{array}$$

## Step 3: solving the model

- ▶ Now we want to solve the model

$$\begin{array}{llllll} \max & 500x_1 & + & 400x_2 & + & 200x_3 \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5. \end{array}$$

- ▶ Wait... this problem is **unbounded**.
  - ▶  $(0, 0, 0)$  is feasible and results in \$0 as my revenue.
  - ▶  $(-1, 2, 0)$  is feasible and results in \$300 as my revenue.
  - ▶  $(-2, 4, 0)$  is feasible and results in \$600 as my revenue.
  - ▶ And so on and so on.
- ▶ We will become millionaires! What is wrong here?

## Step 4: testing and revising the model

- ▶ We cannot bring “negative two” textbooks.
- ▶ How about this:

$$\begin{array}{ll} \max & 500x_1 + 400x_2 + 200x_3 \\ \text{s.t.} & 4x_1 + 2x_2 + 3x_3 \leq 5 \\ & x_i \geq 0 \quad \forall i = 1, \dots, 3. \end{array}$$

- ▶ The best solution is  $(0, 2.5, 0)$ . Still wrong!
- ▶ How about this:

$$\begin{array}{ll} \max & 500x_1 + 400x_2 + 200x_3 \\ \text{s.t.} & 4x_1 + 2x_2 + 3x_3 \leq 5 \\ & x_i \geq 0 \quad \forall i = 1, \dots, 3. \\ & x_i \leq 1 \quad \forall i = 1, \dots, 3. \end{array}$$

- ▶ The best solution is  $(0.75, 1, 0)$ . Still wrong!

## Step 4: testing and revising the model

- ▶ What we still need: We cannot split a book:

$$x_i \in \{0, 1\} \quad \forall i = 1, \dots, 3.$$

- ▶ The final formulation:<sup>2</sup>

$$\begin{aligned} \max \quad & 500x_1 + 400x_2 + 200x_3 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + 3x_3 \leq 5 \\ & x_i \in \{0, 1\} \quad \forall i = 1, \dots, 3. \end{aligned}$$

- ▶ The best solution is (0, 1, 1). Makes sense!

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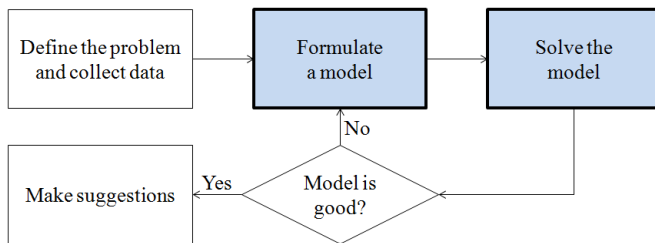
<sup>2</sup>The problem is an example of the **knapsack** problem, one of the most fundamental problem in Computer Science.

## Lastly: Making managerial suggestions

- ▶ “(0, 1, 1)” means nothing to you.
  - ▶ It will also mean nothing to your boss or any manager.
  - ▶ We need **suggestions** on what to do!
  - ▶ We need to **interpret** the solution.
- ▶ Step 5: Given our model and the solution we obtain, we suggest you to sell the textbooks of Computer Programming and OR!
  - ▶ Please do so at least after you pass these courses.

## Summary

- ▶ An OR study is conducted in the following five steps:



- ▶ In this course, we will focus on Steps 2 and 3.
  - ▶ These technical parts require **practices** but no **experience**.
  - ▶ You will do Step 4 by yourselves from time to time.
  - ▶ You will get a taste on Steps 1 and 5 when doing your final project.

## The DFSI principle

- ▶ When you are asked to solve a decision problem in this course, you **MUST** do the following four things:
  - ▶ Step 1: **Define** the decision variables (and the notations you use for parameters).
  - ▶ Step 2: **Formulate** the problem as a mathematical model by writing down the objective function and constraints.
  - ▶ Step 3: **Solve** the model by finding the values for all decision variables in an optimal solution.
  - ▶ Step 4: **Interpret** the optimal solution by indicating “what to do”.



# Agenda

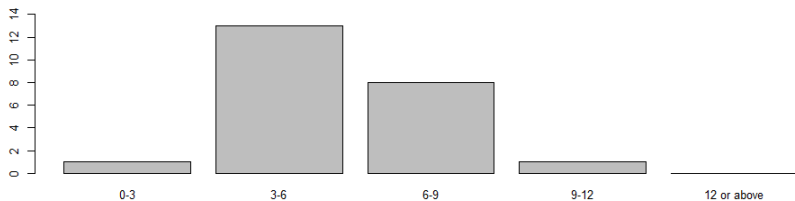
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## Feedback from the previous semester

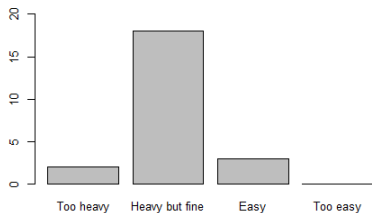
- ▶ About the survey:
  - ▶ A random sample of 23 students through an anonymous online survey.
  - ▶ Between 2015/4/18 and 2015/4/26.
- ▶ The result:

Helpfulness of...	Average (out of 5)
Lecture videos	4.17
Pre-lecture problems	4.09
Lecture problems and in-class discussions	4.17
(Individual) homework	4.08
(Group) case studies	4.00

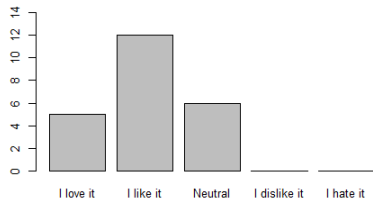
Hours spent per week (including lectures)



How heavy is this course?



Up to today, how do you like (enjoy) this course?



## One student's opinion

*I don't see why each single exercise we do are graded!! We are not babies anymore and don't need to have grade to work, we can also work for our own pleasure of learning new things. And i would like to have exercises before doing so much graded problems, we have no exercises at all to practice before having grades!!! We only have test (pre-lecture problems, in-class problems, homework, case studies etc..!) and i feel more and more demotivated by this course whereas the subject interests me.*

*And I would like to have the lecture video, prelecture problems and homework much more earlier! 3 days is not enough at all, it takes me a lot of time to do it and I have other demanding course!! If we want to be able to organize our time we should have more time to do the exercises and homework.*

*And I said that in-class problems are not helpful because I want to have proper answer. I feel I learn nothing in class because I feel I am working for nothing. And everything I do in class will be forgotten in 5 min because we don't have enough time to think, it's always a competition about time and grades and at the end we don't know If our idea was right or wrong and we can never come back on the exercise to check if 1 month later we are still able or not to redo the exercise.*

## One student's opinion

*I don't want to do OR in my whole life as my job, I am just interested in it and want to have some little knowledge and that's all. I don't want to spend my whole free time to read research papers about OR on the internet to find the answer of the exercises! I would prefer to have answers to understand at least what we see in course. I think that to do the prelecture problems, homework's, case studies and the project is already quite a good investment in the course. I don't need personally to go further than the course because i will never do OR anymore later, I choose this course only for my personal interest to have an idea of what is OR, not more. I mean I want to do exercises and spend time on this subject but not my whole week. Maybe this course is design only for students who want to do research about OR in their future job and want to spend the whole week and week end to study OR but it should be clear at the beginning in this case..*

*The teacher is really nice, loves his subject and invests a lot of time in this course which I appreciate by the way. I don't think the teacher is going to change his method, but as you ask my opinion I just tell what I personally think..*

## So let's make it clear at the beginning

- ▶ This course is a required course for the IM students.
  - ▶ It needs to be technical and difficult.
  - ▶ It covers applications and theories.
- ▶ In 2015:
  - ▶ Students thought that this course is heavy (but fine).
  - ▶ But most of them only spent 0–3 hours at home every week...
- ▶ In 2016:
  - ▶ Most materials and the format will be the same.
  - ▶ Lecture problems will be provided to students after lectures.
- ▶ To take this course, please agree (and be happy) with the above.