Operations Research, Spring 2016

Pre-lecture Problems for Lecture 7: Integer Programming

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Note. The deadline of submitting the pre-lecture problem is ***10:10am, April 7, 2015***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) In this problem, we will use the branch-and-bound algorithm to solve the following IP

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 16 \\ & x_2 \leq 7.5 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2. \end{aligned}$$

   (a) Solve the linear relaxation of the given IP. Show that both x_1 and x_2 are fractional in the optimal solution.

   (b) Branch on x_1 and continue until the IP is fully solved. Depict the branching tree.

   (c) Instead of branch on x_1, branch on x_2 and continue until the IP is fully solved. Depict the branching tree. Compare the result with branching on x_1.

2. (0 point) Consider the following integer program

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 - 6 \leq M_1 z \\ & x_1 + 2x_2 - 8 \leq M_2(1 - z) \\ & x_1 \leq 10 \\ & x_2 \leq 10 \\ & x_i \geq 0 \quad \forall i = 1, 2 \\ & z \in \{0, 1\}, \end{aligned}$$

   where M_1 and M_2 are parameters and x_1, x_2, and z are variables. The binary variable z is to select at least one constraint to be satisfied.

   (a) What values of M_1 and M_2 can enable z to do the "at-least-one" selection?

   (b) Depict the feasible region on the (x_1, x_2) plane. Then graphically solve the IP.
   Hint. The feasible region is nonconvex on the (x_1, x_2) plane. To solve this problem graphically, no branch-and-bound is needed.

3. (10 points) Use the branch-and-bound algorithm to solve the IP

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 6 \\ & 5x_1 + 8x_2 \leq 32 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2. \end{aligned}$$