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Suggested Solution for Pre-lecture Problems for Lecture 8

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1. (a) Let the demand set I and the location set J both be the six towns , who locate at {(0, 60), (20, 50), (30, 20), (40, 80), (50, 50), (90, 60)}.
   Suppose that there are 52 weeks in a year, the 5-year demand size is $h_i \in$ {2600000, 3900000, 3120000, 8000, 2080000, 780000}, at demand $i \in I$.
   The distance between location $j \in J$ and demand $i \in I$ is d_{ij}.
   The fixed construction cost at location $j \in J$ is $f_j \in \{200000, 180000, 160000, 190000, 150000, 200000\}$.
   Let $x_j = 1$ if a DC is built at location $j \in J$ or 0 otherwise.
   Let $y_{ij} = 1$ is demand $i \in I$ is served by DC at location $j \in J$ or 0 otherwise.

$$\begin{aligned}
\min \quad & \sum_{i\in I}\sum_{j\in J}\frac{h_i j}{500} d_{ij} y_{ij} + \sum_{j\in J} f_j x_j \\
\text{s.t.} \quad & y_i j \leq x_j \quad \forall i \in I, j \in J \\
& \sum_{j\in J} y_{ij} = 1 \quad \forall i \in I \\
& x_3 = 1 \\
& x_j \in \{0,1\} \quad \forall j \in J \\
& y_i \in \{0,1\} \quad \forall i \in I, j \in J.
\end{aligned}$$

   (b) Reset the location set J to {(0, 60), (20, 50), (30, 20), (40, 80), (50, 50), (90, 60), (0, 20), (20, 40), (40, 30), (60, 40)} and everything follows.

2. (a) For each job $j \in J = \{1, 2, ..., 10\}$, the processing time is p_j and the due time is d_j , where $p_j \in$ {6, 9, 3, 5, 10, 6, 3, 9, 7, 10} and $d_j \in$ { 50, 53, 55, 56, 59, 60, 62, 67, 68,70}.
   let C_j be the completion time of job $j \in J$.
   Let $z_{ij} = 1$ if job j is before job i or 0 otherwise, $i \in J, j \in J, i < j$.
   Let $M = \sum_{j\in J} p_j$.

$$\begin{aligned}
\min \quad & \sum_{j\in J} T_j \\
\text{s.t.} \quad & T_j \geq C_j - d_j & \forall j \in J \\
& C_i + p_j - C_j \leq M z_{ij} & \forall i \in J, j \in J, i < j \\
& C_j + p_i - C_i \leq M(1 - z_{ij}) & \forall i \in J, j \in J, i < j \\
& T_j \geq 0, C_j \geq 0 & \forall j \in J \\
& z_{ij} \in \{0,1\} & \forall i \in J, j \in J, i < j.
\end{aligned}$$

(b)

$$
\begin{array}{lll}
\min & \sum_{j \in J} T_j & \\
\text{s.t.} & T_j \geq C_j - d_j & \forall j \in J \\
& C_i + p_j - C_j \leq M z_{ij} & \forall i \in J, j \in J, i < j \\
& C_j + p_i - C_i \leq M(1 - z_{ij}) & \forall i \in J, j \in J, i < j \\
& T_j \geq 0, C_j \geq 0 & \forall j \in J \\
& z_{15} = 0 & \\
& z_{34} = 0 & \\
& z_{57} = 0 & \\
& z_{67} = 0 & \\
& z_{69} = 0 & \\
& z_{6(10)} = 0 & \\
& z_{ij} \in \{0, 1\} & \forall i \in J, j \in J, i < j.
\end{array}
$$

3. (a) Following the notation in Problem 1, let $a_{ij} = 1$ if $d_{ij} < 40$ or 0 otherwise, $i \in I$, $j \in J$.

$$
\begin{array}{lll}
\min & \sum_{j \in J} x_j & \\
\text{s.t.} & \sum_{j \in J} a_{ij} x_j \geq 1 & \forall i \in I \\
& x_3 = 1 & \\
& x_j \in \{0, 1\} & \forall j \in J.
\end{array}
$$

(b) Let V be the 6 towns.
Let E be all the paths between 6 towns.
Let d_{ij} be the distance of path $(i, j) \in E$.
Let $x_{ij} = 1$ if the path between the town i and the town j is selected, $(i, j) \in E$.

$$
\begin{array}{lll}
\min & \sum_{(i,j) \in E} d_{ij} x_{ij} & \\
\text{s.t.} & \sum_{i \in V, i \neq k} x_{ik} = 1 & \forall k \in V \\
& \sum_{j \in V, j \neq k} x_{kj} = 1 & \forall k \in V \\
& \sum_{j \in S, j \in S, i \neq j} x_{ij} \leq |S| - 1 & \forall S \subsetneq V, |S| \geq 2 \\
& x_{ij} \in \{0, 1\} & \forall (i, j) \in E.
\end{array}
$$