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Suggested Solution For Case 2

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1. Let the decision variables be

$$x_{ij} = \text{the number of products transferred from DC at location } i \text{ to store at location } j.$$

$$b_i = \begin{cases} 1 & \text{build DC at location } i \\ 0 & \text{otherwise.} \end{cases}, i = 1, ..., 10$$

Let the parameters be

$$\begin{aligned} S_j &= \text{the number of products demanded by store at location } j, \\ M_i &= \text{cost per product of maintaining the operation in DC at location } i, \\ K_i &= \text{maximum scale of DC at location } i, \text{ and} \\ D_{ij} &= \text{distant between DC at location } i \text{ and store at location } j. \\ C_i &= \text{construction cost of store } i \text{ .} \end{aligned}$$

The formulation is

$$\begin{aligned} \min \quad & \sum_{i=1}^{10} \left(M_i \sum_{j=1}^{100} x_{ij} \right) + \sum_{i=1}^{10} \sum_{j=1}^{100} D_{ij} x_{ij} + \sum_{i=1}^{10} b_i C_i \\ \text{s.t.} \quad & \sum_{i=1}^{10} x_{ij} = S_j \quad \forall j = 1, ..., 100 \\ & \sum_{j=1}^{100} x_{ij} \leq K_i b_i \quad \forall i = 1, ..., 10 \\ & x_{ij} \geq 0 \quad \forall i = 1, ..., 10 \quad \forall j = 1, ..., 100. \end{aligned}$$

- Objective function: Adding maintenance cost, replenishment cost,and construction cost.
- Constraint 1: All the demand of each store must be exactly satisfied.
- Constraint 2: The scale of each DC cannot exceed its maximum scale.
- Constraint 3: The number of products transferred must be nonnegative.

2. AMPL model is shown below and the AMPL data is the same as case 1.

```
param D; #10
param S; #100

param Maintenance{i in 1..D};
param Demand{j in 1..S};
param Scale{j in 1..S};
param ConstructionCost{i in 1..D};
param Distance{i in 1..D, j in 1..S};

var x{i in 1..D, j in 1..S};
var Settle{i in 1..D} binary;

minimize cost:
```

```
sum{i in 1..D}( Maintenance[i] *
sum{j in 1..S}(x[i, j]))+
sum{i in 1..D, j in 1..S} (Distance[i, j] * x[i, j])+
sum{i in 1..D}(ConstructionCost[i] * Settle[i]);

subject to demandConstraint{j in 1..S}:
sum{i in 1..D}(x[i, j]) = Demand[j];
subject to ScaleConstraint{i in 1..D}:
sum{j in 1..S}(x[i, j]) <=Settle[i]*Scale[i];
subject to nonnegX{i in 1..D, j in 1..S}:
x[i, j] >= 0;
```

3. The objective value is 163770. Table 1 shows the optimal construction and scale of DCs. Table 1 is the optimal scale of each DC.

DC	1	6	7	9	10
Total	1517	1222	443	524	766

Table 1: optimal scale of each DC

Table 2 is the replenishment plan

S \ D	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	53	0	0	0	0
2	0	0	0	0	0	79	0	0	0	0
3	0	0	0	0	0	34	0	0	0	0
4	0	0	0	0	0	29	0	0	0	0
5	0	0	0	0	0	48	0	0	0	0
6	0	0	0	0	0	0	0	51	0	0
7	0	0	0	0	0	0	0	0	0	34
8	0	0	0	0	0	11	0	0	0	0
9	0	0	0	0	0	0	0	0	96	0
10	40	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	13	0	0	0
12	0	0	0	0	0	0	0	0	0	19
13	0	0	0	0	0	0	0	0	0	5
14	0	0	0	0	0	0	0	0	0	90
15	0	0	0	0	0	0	0	0	0	4
16	86	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	81
18	0	0	0	0	0	0	86	0	0	0
19	0	0	0	0	0	49	0	0	0	0
20	0	0	0	0	0	0	0	0	0	29
21	0	0	0	0	0	0	0	0	0	92
22	54	0	0	0	0	0	0	0	0	0
23	14	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	63	0	0	0
25	0	0	0	0	0	54	0	0	0	0
26	0	0	0	0	0	0	0	0	56	0
27	0	0	0	0	0	90	0	0	0	0
28	0	0	0	0	0	22	0	0	0	0
29	0	0	0	0	0	0	0	0	0	1
30	0	0	0	0	0	27	0	0	0	0
31	35	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	89

33	36	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	32	0	0	0	0
35	20	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	41
37	26	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	17
39	0	0	0	0	0	0	0	0	0	67
40	0	0	0	0	0	0	0	0	0	49
41	0	0	0	0	0	54	0	0	0	0
42	0	0	0	0	0	0	28	0	0	0
43	0	0	0	0	0	0	0	0	0	76
44	0	0	0	0	0	0	0	0	93	0
45	0	0	0	0	0	79	0	0	0	0
46	0	0	0	0	0	0	0	0	91	0
47	0	0	0	0	0	0	44	0	0	0
48	47	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	47	0
50	0	0	0	0	0	0	7	0	0	0
51	0	0	0	0	0	52	0	0	0	0
52	0	0	0	0	0	0	0	0	0	8
53	0	0	0	0	0	28	0	0	0	0
54	87	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	46	0	0	0	0
56	0	0	0	0	0	0	0	0	0	25
57	0	0	0	0	0	66	0	0	0	0
58	43	0	0	0	0	0	0	0	0	0
59	14	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	88	0	0	0	0
61	0	0	0	0	0	0	0	0	22	0
62	0	0	0	0	0	0	0	0	62	0
63	15	0	0	0	0	0	0	0	0	0
64	17	0	0	0	0	0	0	0	0	0
65	77	0	0	0	0	0	0	0	0	0
66	78	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	50	0	0	0	0
68	16	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	34	0	0	0
70	95	0	0	0	0	0	0	0	0	0
71	44	0	0	0	0	0	0	0	0	0
72	53	0	0	0	0	0	0	0	0	0
73	41	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	7
75	0	0	0	0	0	0	23	0	0	0
76	28	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	7
78	57	0	0	0	0	0	0	0	0	0
79	58	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	99	0	0	0	0
81	51	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	28	0	0	0	0
83	0	0	0	0	0	13	0	0	0	0
84	0	0	0	0	0	0	0	0	0	25
85	0	0	0	0	0	43	0	0	0	0
86	19	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	17	0
88	52	0	0	0	0	0	0	0	0	0

89	0	0	0	0	0	0	0	0	33	0
90	0	0	0	0	0	0	0	79	0	0
91	89	0	0	0	0	0	0	0	0	0
92	59	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	3	0	0	0	0
94	0	0	0	0	0	45	0	0	0	0
95	7	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	7	0
97	1	0	0	0	0	0	0	0	0	0
98	58	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	15	0	0
100	100	0	0	0	0	0	0	0	0	0
Total	1517	0	0	0	0	1222	443	0	524	766

Table 2: Replenishment plan

Figure 1 is the scatter plot of the replenishment plan.

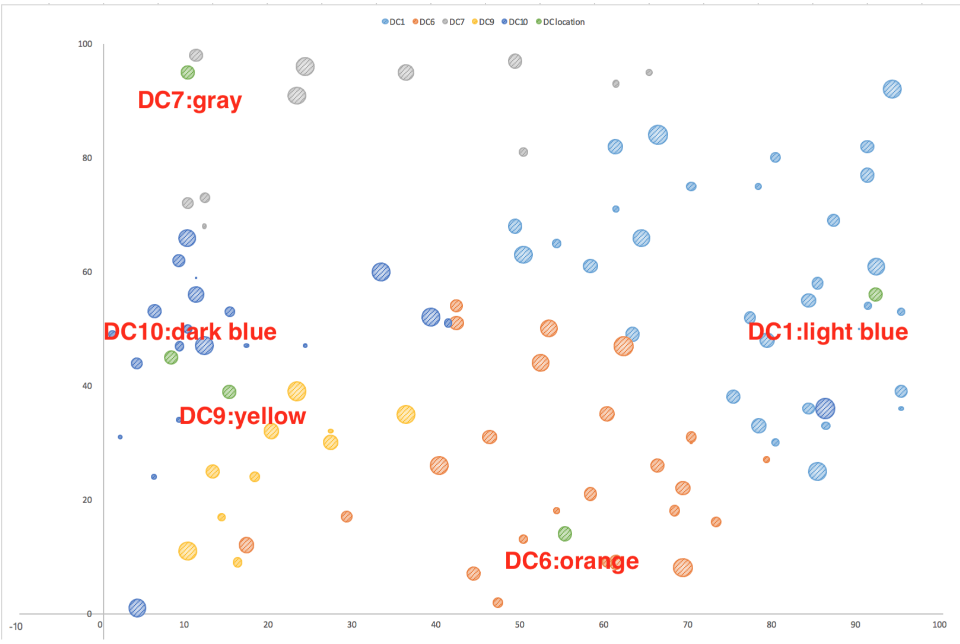

Figure 1: Replenishment plan.

4. Let the decision variables be

x_{ij} = the number of products transferred from DC at location i to store at location j.

$$b_i = \begin{cases} 1 & \text{build DC } i \\ 0 & \text{otherwise.} \end{cases}, i = 1, ..., 10$$

$$z_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 40. \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij} = x_{ij} z_{ij}$$

Let the parameters be

$$
\begin{aligned}
S_j &= \text{the number of products demanded by store at location } j, \\
M_i &= \text{cost per product of maintaining the operation in DC at location } i, \\
K_i &= \text{maximum scale of DC at location } i, \text{ and} \\
D_{ij} &= \text{distant between DC at location } i \text{ and store at location } j. \\
C_i &= \text{construction cost of store } i \text{ .}
\end{aligned}
$$

The formulation is

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{10}\left(M_i \sum_{j=1}^{100} x_{ij}\right) + \sum_{i=1}^{10}\sum_{j=1}^{100} D_{ij}\Big\{z_{ij}\Big[0.05(x_{ij}-40)+40\Big] + (1-z_{ij})x_{ij}\Big\} + \sum_{i=1}^{10} b_i C_i = \\
& \sum_{i=1}^{10}\left(M_i \sum_{j=1}^{100} x_{ij}\right) + \sum_{i=1}^{10}\sum_{j=1}^{100} D_{ij}(x_{ij} - 0.95y_{ij} + 38z_{ij}) + \sum_{i=1}^{10} b_i C_i \\
\text{s.t.} \quad & \sum_{i=1}^{10} x_{ij} = S_j \quad \forall j = 1, ..., 100 \\
& \sum_{j=1}^{100} x_{ij} \le K_i b_i \quad \forall i = 1, ..., 10 \\
& x_{ij} - 40 \le 1000 z_{ij} \quad \forall i = 1, ..., 10, \forall j = 1, ..., 100 \\
& y_{ij} \le 1000 z_{ij} \quad \forall i = 1, ..., 10, \forall j = 1, ..., 100 \\
& y_{ij} \le x_{ij} + 1000(1 - z_{ij}) \quad \forall i = 1, ..., 10, \forall j = 1, ..., 100 \\
& x_{ij} \ge 0 \quad \forall i = 1, ..., 10 \quad \forall j = 1, ..., 100.
\end{aligned}
$$

- Objective function: Adding maintenance cost, replenishment cost,and construction cost.
- Constraint 1: All the demand of each store must be exactly satisfied.
- Constraint 2: The scale of each DC cannot exceed its maximum scale.
- Constraint 3: Set z_{ij} be 1 if $x_{ij} > 40$ otherwise $z_{ij} = 0$.
- Constraint 4 and 5: Let the function be linear.
- Constraint 6: The number of products transferred must be nonnegative.

5. AMPL model is shown below and the AMPL data is the same as case 1.

```
param D; #10
param S; #100
param a; #xij threshold
param b; #fee

param Maintenance{i in 1..D};
param Demand{j in 1..S};
param Scale{i in 1..D};
param ConstructionCost{i in 1..D};
param Distance{i in 1..D, j in 1..S};

var x{i in 1..D, j in 1..S} integer;
var Settle{i in 1..D} binary;
var w{i in 1..D, j in 1..S} binary;
var y{i in 1..D, j in 1..S} ; #y[i, j] = x[i, j] * w[i, j]

minimize cost:
sum{i in 1..D}( Maintenance[i] * sum{j in 1..S}(x[i, j]))+
```

```
sum{i in 1..D, j in 1..S} (Distance[i,j]*(x[i,j]+(b-1)*y[i,j]+(a*(1-b))*w[i,j]))+
sum{i in 1..D}(ConstructionCost[i] * Settle[i]);


subject to demandConstraint{j in 1..S}:
sum{i in 1..D}(x[i, j]) = Demand[j];
subject to ScaleConstraint{i in 1..D}:
sum{j in 1..S}(x[i, j]) <=Settle[i]*Scale[i];

subject to shippingCost1{i in 1..D, j in 1..S}:
x[i, j] -a<= 1000*w[i,j];
subject to shippingCost2{i in 1..D, j in 1..S}:
y[i, j] <= w[i,j]*200;
subject to shippingCost3{i in 1..D, j in 1..S}:
y[i,j] <= x[i, j] + (1-w[i, j])*200;


subject to nonnegX{i in 1..D, j in 1..S}:
x[i, j] >= 0;
```

The objective value is 123580.85. Table 1 shows the optimal construction and scale of DCs. Table 3 is the optimal scale of each DC.

DC	1	6	7	9	10
Total	1562	1295	443	471	701

Table 3: optimal scale of each DC

Table 4 is the replenishment plan

S \ D	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	53	0	0	0	0
2	0	0	0	0	0	79	0	0	0	0
3	0	0	0	0	0	34	0	0	0	0
4	0	0	0	0	0	29	0	0	0	0
5	0	0	0	0	0	48	0	0	0	0
6	0	0	0	0	0	0	51	0	0	0
7	0	0	0	0	0	0	0	0	0	34
8	0	0	0	0	0	11	0	0	0	0
9	0	0	0	0	0	0	0	0	96	0
10	40	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	13	0	0	0
12	0	0	0	0	0	0	0	0	0	19
13	0	0	0	0	0	0	0	0	0	5
14	0	0	0	0	0	0	0	0	0	90
15	0	0	0	0	0	0	0	0	0	4
16	86	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	81
18	0	0	0	0	0	0	86	0	0	0
19	0	0	0	0	0	49	0	0	0	0
20	0	0	0	0	0	0	0	0	19	10
21	0	0	0	0	0	0	0	0	0	92
22	54	0	0	0	0	0	0	0	0	0
23	14	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	63	0	0	0

25	0	0	0	0	0	54	0	0	0	0
26	0	0	0	0	0	0	0	0	56	0
27	0	0	0	0	0	90	0	0	0	0
28	0	0	0	0	0	22	0	0	0	0
29	0	0	0	0	0	0	0	0	0	1
30	0	0	0	0	0	27	0	0	0	0
31	35	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	89
33	36	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	32	0	0	0	0
35	20	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	41
37	26	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	11	6
39	0	0	0	0	0	0	0	0	0	67
40	0	0	0	0	0	0	0	0	0	49
41	0	0	0	0	0	54	0	0	0	0
42	0	0	0	0	0	0	28	0	0	0
43	0	0	0	0	0	0	0	0	0	76
44	0	0	0	0	0	93	0	0	0	0
45	0	0	0	0	0	79	0	0	0	0
46	0	0	0	0	0	91	0	0	0	0
47	0	0	0	0	0	0	44	0	0	0
48	47	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	47	0
50	0	0	0	0	0	0	7	0	0	0
51	0	0	0	0	0	52	0	0	0	0
52	0	0	0	0	0	0	0	0	0	8
53	0	0	0	0	0	28	0	0	0	0
54	87	0	0	0	0	0	0	0	0	0
55	46	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	25
57	0	0	0	0	0	0	0	0	66	0
58	43	0	0	0	0	0	0	0	0	0
59	14	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	88	0	0	0	0
61	0	0	0	0	0	0	0	0	22	0
62	0	0	0	0	0	0	0	0	62	0
63	15	0	0	0	0	0	0	0	0	0
64	17	0	0	0	0	0	0	0	0	0
65	77	0	0	0	0	0	0	0	0	0
66	78	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	50	0	0	0	0
68	15	0	0	0	0	1	0	0	0	0
69	0	0	0	0	0	0	34	0	0	0
70	95	0	0	0	0	0	0	0	0	0
71	44	0	0	0	0	0	0	0	0	0
72	53	0	0	0	0	0	0	0	0	0
73	41	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	7
75	0	0	0	0	0	0	23	0	0	0
76	28	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	7
78	57	0	0	0	0	0	0	0	0	0
79	58	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	99	0	0	0	0

81	51	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	28	0	0	0	0
83	0	0	0	0	0	13	0	0	0	0
84	0	0	0	0	0	0	0	0	0	25
85	0	0	0	0	0	43	0	0	0	0
86	19	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	17	0
88	52	0	0	0	0	0	0	0	0	0
89	0	0	0	0	0	0	0	0	33	0
90	0	0	0	0	0	0	79	0	0	0
91	89	0	0	0	0	0	0	0	0	0
92	59	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	3	0	0	0	0
94	0	0	0	0	0	45	0	0	0	0
95	7	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	7	0
97	1	0	0	0	0	0	0	0	0	0
98	58	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	15	0	0	0
100	100	0	0	0	0	0	0	0	0	0
Total	1562	0	0	0	0	1295	443	0	471	701

Table 4: Replenishment plan

Figure 2 is the scatter plot of the replenishment plan.

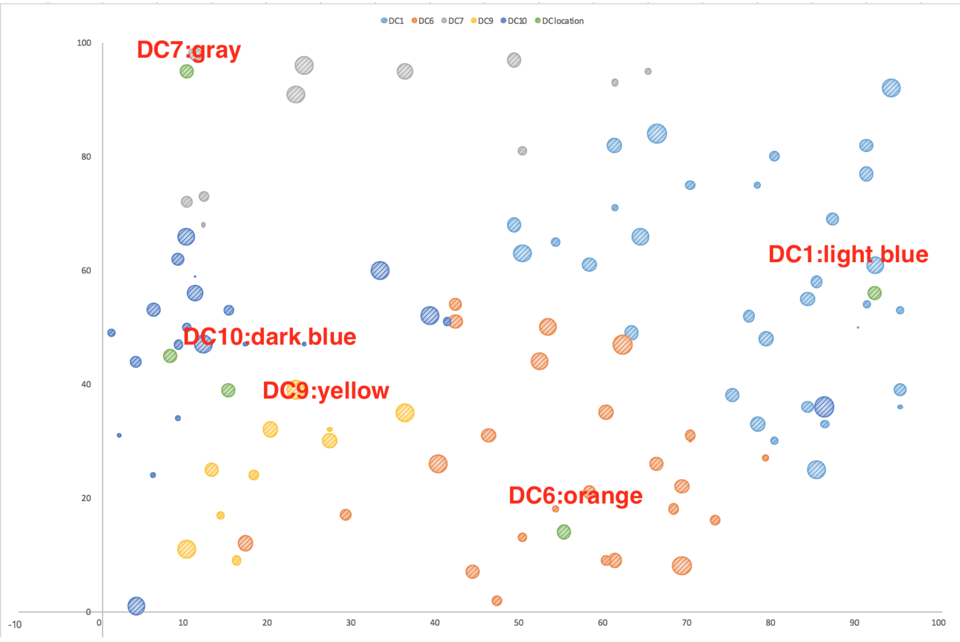

Figure 2: Replenishment plan.