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Suggested Solution for Pre-lecture Problems for Lecture 3

Solution providers: Share Lin
Department of Information Management
National Taiwan University

1. (a) The standard form is

$$\begin{aligned} \max \quad & 5x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 16 \\ & x_1 + 4x_2 + x_4 = 20 \\ & x_2 + x_5 = 8 \\ & x_i \geq 0 \quad \forall i = 1, ..., 5. \end{aligned}$$

(b) Since in the standard form we have five variables and three constraints, there should be three basic variables and two nonbasic variables in a basic solution. There are at most ten basic solutions, which are listed in the table below. The basic feasible solutions are $(\frac{44}{3}, \frac{4}{3}, 0, 0, \frac{20}{3})$, $(16, 0, 0, 4, 8)$, $(0, 5, 11, 0, 3)$, and $(0, 0, 16, 20, 8)$.

x_1	x_2	x_3	x_4	x_5	basis
-12	8	20	0	0	$\{x_1, x_2, x_3\}$
8	8	0	-20	0	$\{x_1, x_2, x_4\}$
$\frac{44}{3}$	$\frac{4}{3}$	0	0	$\frac{20}{3}$	$\{x_1, x_2, x_5\}$
N/A	0	N/A	N/A	0	$\{x_1, x_3, x_4\}$
20	0	-4	0	8	$\{x_1, x_3, x_5\}$
16	0	0	4	8	$\{x_1, x_4, x_5\}$
0	8	8	-12	0	$\{x_2, x_3, x_4\}$
0	5	11	0	3	$\{x_2, x_3, x_5\}$
0	16	0	-44	-8	$\{x_2, x_4, x_5\}$
0	0	16	20	8	$\{x_3, x_4, x_5\}$

(c) The one-to-one mapping between bfs and extreme points is shown in Figure 1.

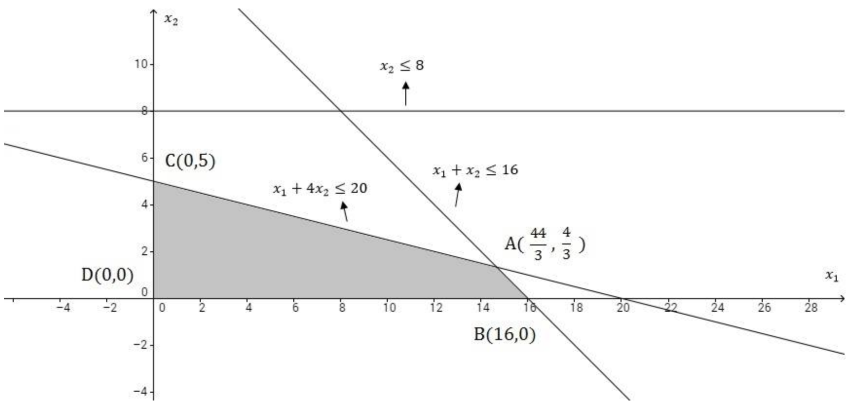

Figure 1: Graphical solution for Problem 1c

2. The initial tableau is

-5	-3	0	0	0	0
1	1	1	1	0	$x_3 = 16$
1	4	0	0	1	$x_4 = 20$
0	1	0	0	1	$x_5 = 8$

We run two iterations to get

-5	-3	0	0	0	0
$\boxed{1}$	1	1	1	0	$x_3 = 16$
1	4	0	0	1	$x_4 = 20$
0	1	0	0	1	$x_5 = 8$

$\rightarrow$

0	2	5	0	0	80
1	1	1	0	0	$x_1 = 16$
0	3	-1	1	0	$x_4 = 4$
0	1	0	0	1	$x_5 = 8$

An optimal solution to the original LP is $(x_1^*, x_2^*) = (16, 0)$ with objective value $z^* = 80$. The route is from $(0,0)$ to $(16,0)$.

3. (a) Let x_1 and x_2 be the number of tables and chairs produced, respectively. The standard form is

$$\begin{aligned} \max \quad & 140x_1 + 100x_2 - 40(3x_1 + 2x_2) \\ \text{s.t.} \quad & 3x_1 + 2x_2 + x_3 = 15 \\ & \frac{1}{0.6}x_1 + x_2 + x_4 = 12 \\ & -2x_1 + x_2 + x_5 = 0 \\ & x_i \geq 0 \quad \forall i = 1, ..., 5. \end{aligned}$$

Since in the standard form we have five variables and three constraints, there should be three basic variables and two nonbasic variables in a basic solution. There are at most ten basic solutions, which are listed in the table below. In this problem, there are eight basic solutions. The basic feasible solutions are $(\frac{15}{7}, \frac{30}{7}, 0, \frac{29}{7}, 0)$, $(0, 0, 15, 12, 0)$, and $(5, 0, 0, \frac{11}{3}, 10)$.

x_1	x_2	x_3	x_4	x_5	basis
$\frac{36}{11}$	$\frac{72}{11}$	$-\frac{87}{11}$	0	0	$\{x_1, x_2, x_3\}$
$\frac{15}{7}$	$\frac{30}{7}$	0	$\frac{29}{7}$	0	$\{x_1, x_2, x_4\}$
27	-33	0	0	87	$\{x_1, x_2, x_5\}$
0	0	15	12	0	$\{x_1, x_3, x_4\}$
$\frac{36}{5}$	0	$-\frac{33}{5}$	0	$\frac{72}{5}$	$\{x_1, x_3, x_5\}$
5	0	0	$\frac{11}{3}$	10	$\{x_1, x_4, x_5\}$
0	0	15	12	0	$\{x_2, x_3, x_4\}$
0	12	-9	0	-12	$\{x_2, x_3, x_5\}$
0	$\frac{15}{2}$	0	$\frac{9}{2}$	$-\frac{15}{2}$	$\{x_2, x_4, x_5\}$
0	0	15	12	0	$\{x_3, x_4, x_5\}$

(b) The initial tableau is

-20	-20	0	0	0	0
$\boxed{3}$	2	1	0	0	$x_3 = 15$
$\frac{5}{3}$	1	0	1	0	$x_4 = 12$
-2	1	0	0	1	$x_5 = 0$

By using the simplex method, we get

$$\begin{array}{ccccc|c}
0 & -\frac{20}{3} & \frac{20}{3} & 0 & 0 & 100 \\
\hline
1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & x_1 = 5 \\
0 & -\frac{1}{9} & -\frac{5}{9} & 1 & 0 & x_4 = \frac{11}{3} \\
0 & \boxed{\frac{7}{3}} & \frac{2}{3} & 0 & 1 & x_5 = 10
\end{array}$$

$$\begin{array}{ccccc|c}
0 & 0 & \frac{60}{7} & 0 & \frac{20}{7} & \frac{900}{7} \\
\hline
1 & 0 & \frac{1}{7} & 0 & -\frac{2}{7} & x_1 = \frac{15}{7} \\
0 & 0 & -\frac{11}{21} & 1 & \frac{1}{21} & x_4 = \frac{29}{7} \\
0 & 1 & \frac{2}{7} & 0 & \frac{3}{7} & x_2 = \frac{30}{7}
\end{array}$$

An optimal solution to the original LP is $(x_1^*, x_2^*) = (\frac{15}{7}, \frac{30}{7})$ with objective value $z^* = \frac{900}{7}$.