Operations Research, Spring 2017

Pre-lecture Problems for Lecture 5: Linear Programming Duality

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Note. The deadline of submitting the pre-lecture problem is ***9:20am, March 23, 2017***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Find the dual for the following LP:

$$\begin{array}{rrcrcrcr}
\max & 4x_1 & - & 2x_2 & + & x_3 & & \\
\text{s.t.} & 2x_1 & + & x_2 & & & \leq & 10 \\
 & & & x_2 & + & x_3 & \geq & 16 \\
 & x_1 & & 3x_2 & - & 3x_3 & = & 14 \\
 & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \leq 0,\ x_3 \text{ urs.}}
\end{array}$$

2. (0 point) Consider a primal LP

$$\begin{array}{rl}
\max & 3x_1 + 5x_2 \\
\text{s.t.} & x_1 + x_2 \leq 8 \\
 & x_1 + 2x_2 \leq 12 \\
 & x_1 \geq 0, x_2 \geq 0.
\end{array}$$

   (a) Find a primal optimal solution $\bar{x}$.
   (b) Formulate the dual LP.
   (c) Solve the dual LP to get a dual optimal solution $\bar{y}$. Show that $c^{\mathrm{T}}\bar{x} = \bar{y}^{\mathrm{T}}b$, where c and b are the primal and dual objective function.

3. (10 points) Consider a primal LP

$$\begin{array}{rl}
\max & 5x_1 + 3x_2 \\
\text{s.t.} & x_1 + x_2 \leq 12 \\
 & x_1 + 2x_2 \leq 16 \\
 & x_1 \geq 0, x_2 \geq 0.
\end{array}$$

   (a) Find a primal optimal solution $\bar{x}$ by the simplex method.
   (b) Formulate the dual LP.
   (c) Solve the dual LP in any way you like to get a dual optimal solution $\bar{y}$. Show that $c^{\mathrm{T}}\bar{x} = \bar{y}^{\mathrm{T}}b$, where c and b are the primal and dual objective function.
   (d) Use the primal optimal basis B you found in Part (a) to verify that $c_B^{\mathrm{T}}A_B^{-1} = \bar{y}^{\mathrm{T}}$.
   (e) Find the shadow prices for the two primal constraints.[1]

[1] If you are applying the correct concept, you may need no calculation for finding them. But maybe you would like to solve two modified primal LPs to do a verification.