Operations Research, Spring 2017

Lecture 2: Introduction to Linear Programming

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. IEDO Chemical manufactures four chemicals: A, B, C, and D. These chemicals are produced via two production processes. Running process 1 for an hour costs \$7 and yields 3 units of A, 2 of B, 1 of C, and 1 of D. Running process 2 for an hour costs \$3 and produces 1 unit of A and 1 of B. To meet customer demands, at least 12 units of A, 6 of B, 4 of C, and 3 of D must be produced daily.

   (a) (3 points) Formulate an LP to determine a daily production plan that minimizes the cost of meeting IEDO Chemicals daily demands.

   (b) (3 points) Graphically solve the LP.

2. (3 points) Graphically solve the following LP:

$$
\begin{aligned}
\min \quad & x_1 - x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 6 \\
& x_1 - x_2 \geq 0 \\
& -2x_1 + x_2 \geq 6 \\
& x_1 \geq 0, x_2 \geq 0.
\end{aligned}
$$

3. Consider the following LP:

$$\begin{aligned} \max \quad & 2x_1 - x_2 \\ \text{s.t.} \quad & 8x_1 - 4x_2 \leq 16 \\ & 3x_1 - 4x_2 \leq 12 \\ & x_1 \geq 0, x_2 \leq 0. \end{aligned}$$

(a) (3 points) Graphically determine whether it has an unique optimal solution, has multiple optimal solutions, is infeasible, or is unbounded.

(b) (2 points) Find the binding constraints, if any, at $(x_1, x_2) = (2, 0)$.

(c) (2 points) Find the binding constraints, if any, at $(x_1, x_2) = (1, 2)$.

(d) (2 points) For each optimal solution, find the binding constraints at it, if any.

4. For each set below, find the set of extreme points.

(a) (2 points) The triangle whose vertices are $(1,0)$, $(0,1)$, and $(1,1)$.

(b) (2 points) $\{x \in \mathbb{R}^2 | x_1^2 + x_2^2 \leq 1, x_1 + x_2 \geq -1\}$.

(c) (2 points) $\{1, 2, 3, 4, 4.5, 5\}$.

(d) (2 points) $\{x \in \mathbb{R}^n | x_i \geq 0 \quad \forall i = 1, ..., n\}$.

(e) (2 points) $\{A \in \mathbb{R}^{n \times n} | A = A^{\mathrm{T}}\}$.

5. (6 points) IEDO Oil has refineries in Kaohsiung and Taipei. Currently, the Kaohsiung refinery can refine up to 3 million barrels of oil per year, and the Taipei refinery up to 4 million. Once refined, oil is shipped to two distribution points: Hsinchu and Taichung. IEDO Oil estimates that each distribution point can sell up to 6 million barrels per year. Because of differences in shipping and refining costs, the profit earned per million barrels of oil shipped depends on where the oil was refined and on the point of distribution:

From	To Hsinchu	To Taichung
Kaohsiung	\$16,000	\$19,000
Taipei	\$22,000	\$18,000

Formulate an LP that maximizes IEDO's profits for the next year. Intuitively solve the LP.

6. (6 points) Following from the previous problem, IEDO Oil is now considering expanding the capacity of each refinery. Each million barrels of annual refining capacity that is added will cost \$140,000 for the Kaohsiung refinery and \$180,000 for the Taipei refinery. Capacity can only be added now but can be used in the future ten years. Formulate an LP that maximizes IEDO's profits less expansion costs over a ten-year period.

7. (6 points) Mary uses chemicals 1 and 2 to produce two drugs. Drug 1 must be made of at least 60% chemical 1. For example, blending 11 oz of chemical 1 and 9 oz of chemical 2 makes there only $\frac{11}{11+9} = 55\%$ of chemical 1, which does not make the produced drug 1 of the required quality. For drug 2, there must be at least 50% chemical 2. Up to 100 oz of drug 1 can be sold at \$6 per oz; up to 90 oz of drug 2 can be sold at \$5 per oz. Up to 130 oz of chemical 1 can be purchased at \$6 per oz, and up to 80 oz of chemical 2 can be purchased at \$4 per oz. Formulate an LP that can be used to maximize Mary's profits.

   Note. Is your formulation linear or nonlinear? Double check!

8. (6 points) Mary uses chemicals 1, 2, ..., n to produce drugs 1, 2, ..., m. Drug i must be made of at least $100M_{ij}\%$ chemical j (e.g., if $M_{ij} = 0.5$, that means drug i must be made of at least one half of chemical j). Up to D_i oz of drug i can be sold at $\$P_i$ per oz. Up to S_j oz of chemical j can be purchased at $\$C_j$ per oz. Formulate an LP that can be used to maximize Mary's profits.