Statistics and Data Analysis

Supplements for Hypothesis Testing

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Steps of hypothesis testing

- To conduct a test, go through the following four steps:
  - **Hypothesis**: Write down H_0 and H_a.
  - **Test**: Select an appropriate test (z test, t test, etc.) to apply.
  - **Calculation**: Statistics, critical values, and/or p-values obtained by software.
  - **Decision and implication**: Reject or do not reject H_0? What does that mean?
- In this set of slides, we offer you some more examples and explanations of hypothesis testing.
- These materials are **supplemental**:
  - Materials only contained here will not appear in homework or exams.
  - If you can read them by yourself, awesome! If you cannot, it is fine.
  - For the examples, focus on the concepts rather than calculations.
  - Do not ask the instructor to solve the examples for you. However, asking him to clarify some concepts is welcome.

Road map

- **Testing population mean: variance known**.
- Testing population mean: variance unknown.
- Testing population proportion.

Testing the population mean

- There are many situations to test the **population mean** μ.
  - Is the average monthly salary of fresh college graduates above $22,000 (22K)?
  - Is the average thickness of a plastic bottle 2.4 mm?
  - Is the average age of consumers of a restaurant below 40?
  - Is the average amount of time spent on information system projects above six months?
- We will use hypothesis testing to test the population mean.
- Main factor:
  - Whether the **population variance** σ^2 is known.
  - Whether the population is normal.
  - Whether the sample size is large.

Testing the population mean

- When the population variance σ^2 is know:
  - If the population is normal or the sample size $n \geq 30$: z **test**.
  - Otherwise: Nonparametric methods (beyond the scope of this course).
- When the population variance σ^2 is unknown:
  - If the population is normal: t **test**.
  - If the sample size $n \geq 30$: t **test or** z **test**.
  - Otherwise: Nonparametric methods (beyond the scope of this course).

Example 1

- A retail chain has been operated for many years.
- The average amount of money spent by a consumer is \$60.
- A new marketing policy has been proposed: Once a consumer spends \$70, she/he can get one credit. With ten credits, she/he can get one toy for free.
- After the new policy has been adopted for several months, the manager asks: Has the average amount of money spent by a consumer increased? Let $\alpha = 0.01$.
  - Let μ be the average expenditure (in \$) per consumer after the policy is adopted. Is $\mu > 60$?
  - The population standard deviation is \$16.

Example 1: hypothesis and test

- The hypothesis is

$$\begin{aligned} \mathrm{H}_0 &: \mu = 60 \\ \mathrm{H}_a &: \mu > 60. \end{aligned}$$

  - $\mu = 60$ is our **default position**.
  - We want to know whether the population mean **has increased**.
- Some researchers write

$$\begin{aligned} \mathrm{H}_0 &: \mu \leq 60 \\ \mathrm{H}_a &: \mu > 60. \end{aligned}$$

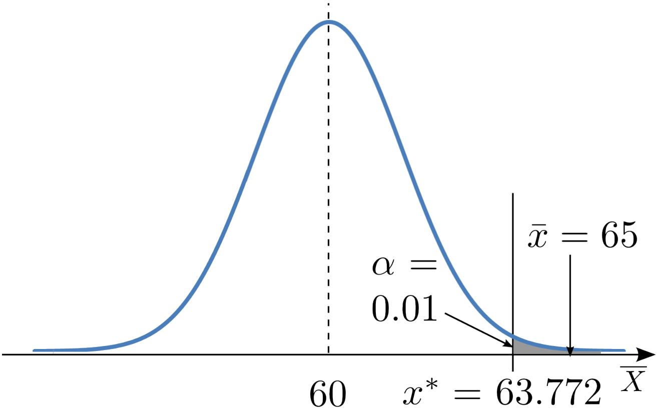
- Because the population variance is known and the sample size is large, we should use the z test.

Example 1: calculation and interpretation

- The manager collects a sample with 100 purchasing records of consumers. The observed sample mean is $\bar{x} = 65$.
- As
$$p\text{-value} = \Pr(\overline{X} \geq 65 | \mu = 60) = 0.000899 < 0.01 = \alpha,$$
we reject H_0.
- With a 99% confidence, the population mean is greater than 60.
- The new marketing policy ($70 for one credit and ten credits for one toy) is successful: Each consumer is willing to pay more (in expectation) under the new policy.

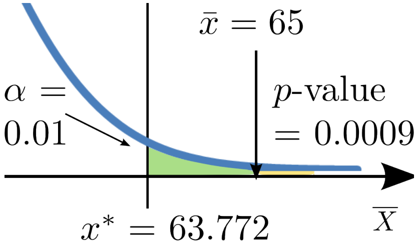
Example 1: graphical illustration

- Because $\bar{x} = 65$ falls in the rejection region $(63.722, \infty)$, we reject the null hypothesis.



Example 1: graphical illustration

- Because p-value $= 0.000899 < 0.01 = \alpha$, we reject the null hypothesis.



Road map

- Testing population mean: variance known.
- **Testing population mean: variance unknown**.
- Testing population proportion.

Example 2

- An MBA program seldom admits applicants without a work experience longer than two years.
- To test whether the average work year of admitted students is above two years, 20 admitted applicants are randomly selected.
- Their work experiences prior to entering the program are recorded.
  - Prior to entering the program, they have an average work experience of 2.5 years. This is the sample mean.
  - The sample standard deviation is 1.3765 years.
- The population is believed to be normal.
- The confidence level is set to 95%.

Example 2: hypothesis

- Suppose the one asking the question is a potential applicant with one year of work experience. He is **pessimistic** and will apply for the program **only if** the average work experience is proven to be **less** than two years.
- The hypothesis is

$$\mathrm{H}_0 : \mu = 2$$
$$\mathrm{H}_a : \mu < 2.$$

  - μ is the average work experience (in years) of all admitted applicants prior to entering the program.
  - To **encourage** him, we need to give him a strong evidence showing that his chance is high.

Example 2: hypothesis and test

- Suppose he is **optimistic** and will not apply for the program **only if** the average work experience is proven to be **greater** than two.
- The hypothesis becomes

$$\begin{aligned} \mathrm{H}_0 &: \mu = 2 \\ \mathrm{H}_\mathrm{a} &: \mu > 2. \end{aligned}$$

  - To **discourage** him, we need to give him a strong evidence showing that his chance is slim.
- Let's consider the optimistic candidate (and $\mathrm{H}_\mathrm{a} : \mu > 2$) first.
- Because the population variance is unknown and the population is normal, we may use the t test.

Example 2A: calculation and interpretation

- Calculation:
  - The p-value is $\Pr(\overline{X} > 2.5 | \mu = 2) = 0.0604$.[1]
- Conclusion:
  - For this one-tailed test, as the p-value $> 0.05 = \alpha$, we do not reject H_0.
  - There is **no strong evidence** showing that the average work experience is longer than two years.
  - The result is not strong enough to discourage the potential applicant, who has only one year of work experience.
- Decision:
  - The (optimistic) applicant **should** apply.

[1] The calculation depends on the t distribution. You do not need to know how to do the calculation.

Example 2B – a pessimistic applicant

- Suppose the applicant is pessimistic and the hypothesis is

$$H_0: \mu = 2$$
$$H_a: \mu < 2.$$

  - The p-value will be $\Pr(\overline{X} < 2.5 | \mu = 2) = 1 - 0.0604 = 0.9396$.
  - We do not reject H_0 and cannot conclude that $\mu < 2$. There is no strong evidence to encourage him.
  - He **should not** apply.
- Note that when we write different alternative hypotheses, the final decision is different!
  - This happens if and only if in both cases we do not reject H_0.

Road map

- Preparations.
- Testing population mean: variance known.
- Testing population mean: variance unknown.
- **Testing population proportion**.

Testing the population proportion

- In many situations, we need to test the **population proportion**.
  - The defective rate or yield rate of a production system.
  - The proportion of people supporting a candidate.
  - The proportion of people supporting a policy.
  - The proportion of people viewing a product web page that will really buy the product (conversion rate).
- How to test the population proportion?
- Suppose we want to test the proportion of male users:
  - Let's label a male user by 1 and non-male users by 0.
  - The population proportion $p = \frac{\sum_{i=1}^{N} x_i}{N}$ is the **population mean**.
  - A sample proportion $\hat{p} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the sample mean.
  - We may apply **the z test** to test population proportion.[2]
- Technical restrictions: $n \geq 30$, $n\hat{p} \geq 5$, and $n(1-\hat{p}) \geq 5$.

[2] We may derive the population standard deviation σ from p as $\sqrt{p(1-p)}$.

The hypotheses

- The population proportion is denoted as p.
- A two-tailed test for the population proportion is

$$H_0 : p = p_0$$
$$H_a : p \neq p_0,$$

where p_0 is the **hypothesized proportion**.

- In a one-tailed test, the alternative hypothesis may be either

$$H_a : p > p_0$$

or

$$H_a : p < p_0.$$

Example 3

- In a factory, it seems to us that the defective rate of our product is too high. Ideally it should be below 1% but some workers believe that it is above 1%.
- If the defective rate is above 1%, we should fix the machine. Otherwise, we do not do anything.
- Let p be the defective rate, the hypothesis is

$$\mathrm{H}_0: p = 0.01$$
$$\mathrm{H}_\mathrm{a}: p > 0.01.$$

- When to adopt $\mathrm{H}_\mathrm{a}: p < 0.01$?

Example 3

- In several random production runs, we found that out of 1000 produced items, 14 of them are defective.
  - The observed sample proportion $\hat{p} = 0.014$.
  - All the technical requirements are satisfied; $n = 1000$, $n\hat{p} = 14$, and $n(1 - \hat{p}) = 986$.
- Suppose the significance level is set of $\alpha = 0.05$, what is our conclusion?

Example 3: calculation and interpretation

- Calculation and conclusion:
  - For this one-tailed test, as

$$\begin{aligned} p\text{-value} &= \Pr(\hat{p} > 0.014 | p = 0.01) \\ &= 0.1018 > 0.05 = \alpha, \end{aligned}$$

    we do not reject H_0.
  - There is **no strong evidence** showing that the defective rate is higher than 1%.
- Decision:
  - We should not try to fix the machine.