Statistics and Data Analysis

Descriptive Statistics (1): Visualization

Ling-Chieh Kung

Department of Information Management
National Taiwan University

Visualizing the data

- We will introduce some common ways to **summarize** a set of data.
  - By **graphs**.
  - By **statistics**.
- This is always the **first step** of any data analysis project: To get intuitions that guide our directions.

Road map

- **Frequency distributions**.
- Quantitative data graphs.
- Qualitative data graphs.
- Visualizing two variables.

Descriptive Statistics

- Consider the column "cnt" in the sheet "Day" of the Excel file "Bike.xlsx".
  - Each number is the number of rentals in a day.
  - 985, 801, 1349, 1562, 1600, 1606, 1510, 959, 822, 1321, 1263, 1162, 1406, 1421, 1248, 1204, 1000, 683, 1650, 1927, ..., and 2729.
- To get more ideas about this data set, we do **Descriptive Statistics**.
  - Sometimes called **exploratory data analysis**.
  - Using data graphs to visualize data or using numbers to summarize data.

Frequency distributions

- The original 731 numbers form a set of **ungrouped data**.
- When data are ungrouped, visualizing them is hard.
- We start by **grouping** them into a **frequency distribution**.
  - Grouped data presented in the form of class intervals and frequencies.
- Let's create an intuitive frequency distribution.

Frequency distributions: an example

- Let's group the daily bike rental data into a frequency distribution.
  - Let's label these 731 numbers are x_1, x_2, ..., and x_{731}.
- Step 1: Find the **range**:

$$\max_{i=1,\ldots,731}\{x_i\} - \min_{i=1,\ldots,731}\{x_i\} = 8714 - 22 = 8692.$$

- Step 2: Let's divide the range into **classes**:
  - These classes are intervals with equal lengths.
  - A typical number of classes is between **5 and 15**.
  - Let's choose 9, for example.
- Step 3: **Class width** $\geq \frac{8692}{9} \approx 965.78$. Let's try 1000.

Frequency distributions: an example

- The resulting classes:

Class	Class interval	(Which means)
1	$[0, 1000)$	$0 \leq x < 1000$
2	$[1000, 2000)$	$1000 \leq x < 2000$
3	$[2000, 3000)$	$2000 \leq x < 3000$
	$\vdots$	
8	$[7000, 8000)$	$7000 \leq x < 8000$
9	$[8000, 9000)$	$8000 \leq x < 9000$

  - How about $[0, 999]$, $[1000, 1999]$, etc.?
  - How about $(0, 1000]$, $(1000, 2000]$, etc.?

Frequency distributions: an example

- Then we **count** to get the frequency distribution at the right.
- This is a set of **grouped data**.
- Some remarks:
  - Typically we have 5 to 15 classes.
  - Typically all classes have the same width.
  - Be aware of class endpoints! Classes should NOT overlap with each other.
  - If there are **outliers**, they should be removed first.

Class interval	Frequency
$[0, 1000)$	18
$[1000, 2000)$	80
$[2000, 3000)$	74
$[3000, 4000)$	107
$[4000, 5000)$	166
$[5000, 6000)$	106
$[6000, 7000)$	86
$[7000, 8000)$	82
$[8000, 9000)$	12

Outliers

- An **outlier** in a data set is a value that is "very weird."
  - May be due to a very rare case.
  - May be due to a typo.
- For examples,
  - A promotion makes the rental free on December 31, 2012. The number of daily rentals is 34231 (originally 2290).
  - One mistakenly typed 654 in January 1, 2011, as 6544.
- Some outliers may be identified with a frequency distribution. Some are not.

Class interval	Frequency
$[0, 1000)$	**17**
$[1000, 2000)$	80
$[2000, 3000)$	**73**
$[3000, 4000)$	107
$[4000, 5000)$	166
$[5000, 6000)$	106
$[6000, 7000)$	**87**
$[7000, 8000)$	82
$[8000, 9000)$	12
$\vdots$	
$[34000, 35000)$	**1**

Something more

- We may add **class midpoints**, **relative frequencies**, and **cumulative frequencies** into a frequency table:

Class interval	Frequency	Class midpoint	Relative frequency	Cumulative frequency
$[0, 1000)$	18	500	2.46%	18
$[1000, 2000)$	80	1500	10.94%	98
$[2000, 3000)$	74	2500	10.12%	172
$[3000, 4000)$	107	3500	14.64%	279
$[4000, 5000)$	166	4500	22.71%	445
$[5000, 6000)$	106	5500	14.50%	551
$[6000, 7000)$	86	6500	11.76%	637
$[7000, 8000)$	82	7500	11.22%	719
$[8000, 9000)$	12	8500	1.64%	731

- How about **cumulative relative frequencies**?

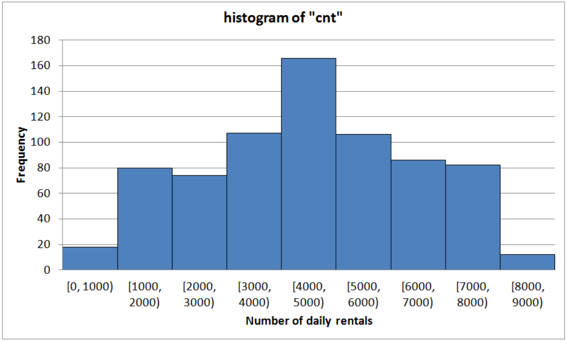
Road map

- Frequency distributions.
- **Quantitative data graphs**.
- Qualitative data graphs.
- Visualizing two variables.

Histograms

- A frequency distribution may be depicted as a **histogram**.

Interval	Freq.
$[0, 1000)$	18
$[1000, 2000)$	80
$[2000, 3000)$	74
$[3000, 4000)$	107
$[4000, 5000)$	166
$[5000, 6000)$	106
$[6000, 7000)$	86
$[7000, 8000)$	82
$[8000, 9000)$	12



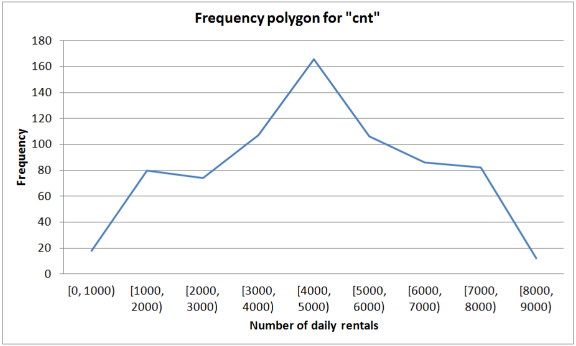
- It consists of a series of **contiguous** rectangles, each representing the frequency in a class.

Histograms

- Histograms may be the most important type of data graphs.
- One particular reason to draw histograms is to get some ideas about the **distribution**.
  - Bell shape? M shape? Skewed?
  - Any outlier?
  - We will discuss distributions in more details.

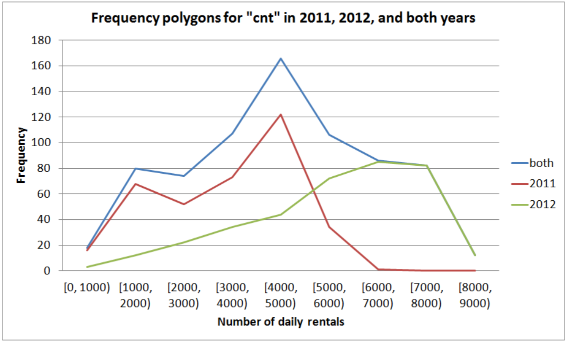
Frequency polygons

- Alternatively, we may draw a **frequency polygon** by using **line segments** connecting dots plotted at class **midpoints**.
  - The information contained in a frequency polygon is quite similar to that contained in a histogram.



Frequency polygons

- It is more convenient to use a frequency polygon to **compare multiple** frequency distributions.

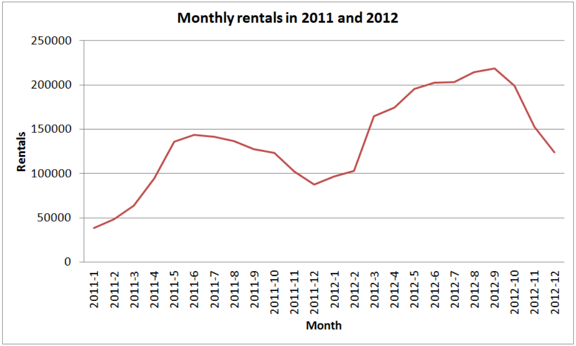


- Both: Uni-modal and symmetric.
- 2011: Bi-modal and skewed to the right (right-tailed).
- 2012: Uni-modal and skewed to the left (left-tailed).

- Warning: People may **misinterpret** a frequency polygon as a **line chart** (for data with a time sequence).

Line charts

- A **line chart** is useful in depicting a time series data.
  - A two-dimensional data set whose first dimension (the x-axis) is for labels of time points.
- It visualizes how a quantity changes as time goes by.
- For our monthly bike rentals:



Road map

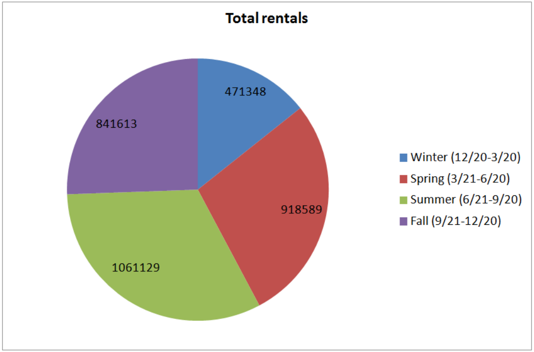
- Frequency distributions.
- Quantitative data graphs.
- **Qualitative data graphs**.
- Visualizing two variables.

Pie charts

- A **pie chart** is a **circular** depiction of data where each slice represents the percentage of the corresponding category.
- It visualizes **relative frequency distributions** well.
- For our bike rental data set:
  - What are the proportions of rentals in the four seasons?
  - What are the proportions of rentals on the seven days of a week?

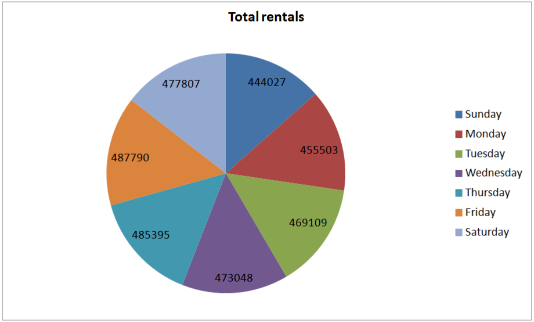
A pie chart for seasonal rentals

Season	Total rentals	Proportion
Winter (12/20-3/20)	471348	14.3%
Spring (3/21-6/20)	918589	27.9%
Summer (6/21-9/20)	1061129	32.2%
Fall (9/21-12/20)	841613	25.6%



A pie chart for rentals among weekdays

Day	Total rentals
Sunday	444027
Monday	455503
Tuesday	469109
Wednesday	473048
Thursday	485395
Friday	487790
Saturday	477807



Data not appropriate for pie charts

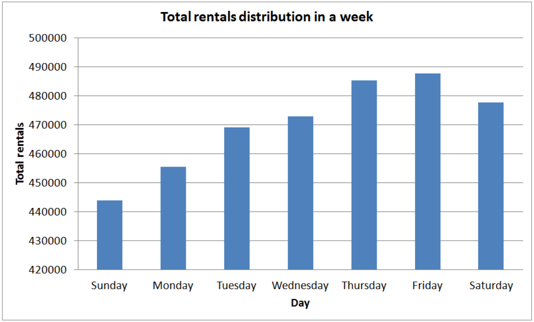
- Pie charts are used to visualize proportions, i.e., subtotals over the overall total.
- It should **not** be used to compare **averages**.
  - The total numbers of rentals made by male and female users are appropriate for a pie chart.
  - The average numbers of rentals per male and female users are not appropriate for a pie chart.

Bar charts

- Pie charts are useful in visualizing the **proportions** of each categories.
- In demonstrating the **differences** among categories, a **bar chart** is a better choice.
  - The larger the category, the longer the bar.
  - Some people draw bars vertically; some horizontally.

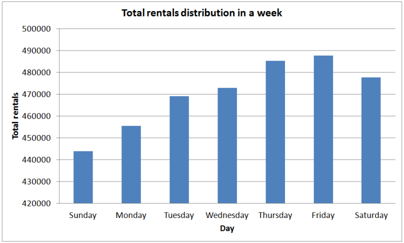
Bar charts

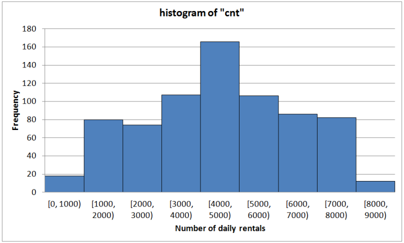
Day	Total rentals
Sunday	444027
Monday	455503
Tuesday	469109
Wednesday	473048
Thursday	485395
Friday	487790
Saturday	477807



Bar charts v.s. histograms

- What are the differences that distinguish a bar chart from a histogram?





  - A bar chart uses **noncontiguous** bars to visualize **categorical** data.
  - A histogram uses **contiguous** bars to visualize **quantitative** data.

Road map

- Frequency distributions.
- Quantitative data graphs.
- Qualitative data graphs.
- **Visualizing two variables**.

Visualizing two variables

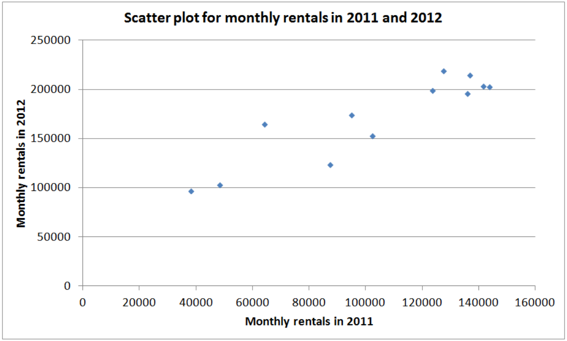
- When we have data for two variables, typically we want to identify whether there is any **relationship** between them.
- Visualizing the data in a two-dimensional manner helps.

Scatter plots

- Sometimes in an observation there are **two** values recorded.
- When the two vales are both measured in quantitative scales, we may depict each observation as a point on a plane to create a **scatter plot**.
- For our bike rental example:
  - How do monthly rentals in 2011 and those in 2012 relate with each other?
  - How do daily casual and registered rentals relate with each other?

Monthly rentals in 2011 and 2012

Month	2011	2012
1	38189	96744
2	48215	103137
3	64045	164875
4	94870	174224
5	135821	195865
6	143512	202830
7	141341	203607
8	136691	214503
9	127418	218573
10	123511	198841
11	102167	152664
12	87323	123713



Daily casual and registered rentals

day	casual	registered
1	331	654
2	131	670
3	120	1229
	⋮	
731	439	2290